# Steganography-Free Zero-Knowledge

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**Abstract.** We revisit the well-studied problem of preventing steganographic communication in multi-party communications. While this is known to be a provably impossible task, we propose a new model that allows circumventing this impossibility. In our model, the parties first publish a single message during an honest *non-interactive* pre-processing phase and then later interact in an execution phase. We show that in this model, it is indeed possible to prevent any steganographic communication in zero-knowledge protocols. Our solutions rely on standard cryptographic assumptions.

## 1 Introduction

Consider the following scenario: a computer at a government agency storing highly classified data has been infected with a stealthy malware. The malware's main purpose is to communicate the classified data to an attacker on the Internet. To minimize the possibility of being detected and quarantined, the malware has been designed to stealthily "encode" the secret data in ordinary communication between the infected computer and the outside world. This may include communication with "honest" entities on the Internet or potentially even the attacker (disguised as an honest user). An intriguing question, which forms the basis of the present work, is whether it is possible to detect such communication?

The above scenario is representative of a broader theme concerning *steganographic communication*, where a party $A$ wants to transmit a secret message to another party $B$ by communicating over a public broadcast channel without being detected by an external *observer* who is listening on the channel. Since the use of an encrypted channel can be easy to detect, $A$ may instead try to embed its message in an innocuous-looking conversation. For example, [39], it may send a photograph of a person to securely transmit bit 0 if the 30th hair from the left is white, and 1 otherwise.

A sequence of works [14, 36, 34, 3] have established that such steganographic communication is always possible in any system with some entropy, and is provably *impossible* to detect. As such, it may seem that the answer to the aforementioned question is negative.

**A New Model for Preventing Steganography.** In this work, we propose a new model for circumventing the aforementioned impossibility result. In our model, any communication (via an interactive protocol) proceeds in two phases: a *non-interactive pre-processing* phase and an *execution* phase. Each party publishes a single message during the pre-processing phase, while the execution phase corresponds to the actual protocol execution. We assume that the parties are honest during the pre-processing phase, but may be completely malicious during the execution phase. *Our main goal is to ensure that any attempts at steganographic communication during the execution phase will be detected by the external observer.*

We, in fact, consider a stronger model where *only one of the parties is required to be honest* during the pre-processing phase. In this case, the malicious parties may be able to subliminally embed information in their pre-processing messages. However, we require that such subliminal communication is limited to the (non-interactive) pre-processing and that no steganographic communication can be performed during the execution phase. Our model is meaningful in our motivating example: if the pre-processing step is executed before the computer is infected, then it ensures that no information can be later leaked by the malware without being detected.

Let us now explain why the pre-processing model can help in preventing steganography. As observed in many prior works, the key source of the problem is that the parties' algorithms may be *randomized*, which opens an avenue for subliminal communication. Removing the use of randomness altogether does not yield a solution since randomness is necessary for most of cryptography [22]. The pre-processing model helps resolve this dilemma. The main insight is that the pre-processing step can be used to "fix" the randomness of the parties, thereby forcing them to become *deterministic* during the execution phase. If the parties deviate from the prescribed strategy, they can be detected by the observer.

A common method to detect deviation from prescribed strategy in any protocol is to use zero-knowledge (ZK) proofs [32], à la Goldreich, Micali, Wigderson (GMW) compiler [29]. However, ZK proofs themselves require randomness [22]. As such, a priori, it might not be clear how to implement the above idea.

### 1.1 Our Contribution

We present a general method for preventing steganographic communication in interactive protocols.

**Defining Steganography Freeness.** We start by defining *steganography freeness* for generic interactive protocols $(S, R)$ in the non-interactive pre-processing model. Intuitively, our notion requires that no adversarial sender $S$ can steganographically communicate even a single bit of information to the receiver $R$ during the execution phase as long as at least one of them was honest during the pre-processing phase. We formalize this via a game-based definition (Section 3) where at the start of the execution phase, the adversarial sender is given a randomly chosen bit $b$. We require that at the end of the execution phase, the probability that the receiver correctly guesses $b$ and the execution transcript is accepted by the observer is only negligibly more than one half.

**Steganography-Free Zero-Knowledge.** Our main tool for achieving steganography freeness in a generic interactive protocol is a new notion of *steganography-free zero-knowledge* (SF-ZK). An SF-ZK argument proceeds in two phases: first, the prover and the verifier participate in a non-interactive pre-processing step where they send a single message to each other. This step is executed *before* the prover receives the statement and the witness. Next, the prover and the verifier participate in the execution phase where the prover proves the validity of the statement.

An SF-ZK argument system must satisfy the standard completeness, soundness, and ZK properties. In particular, soundness (resp. ZK) must hold even if the prover (resp. verifier) is malicious both during the pre-processing as well as the execution phase. Further, SF-ZK must satisfy two new security properties:

- *Observer Soundness:* This property states that for any *false* statement, no coalition of prover and verifier can produce a transcript that will be accepted by the external observer as long as either the prover or the verifier was honest during the pre-processing phase.
- *Computationally Unique Transcripts (CUT):* We define this property w.r.t. languages $\mathcal{L}$ with unique witnesses; however, it can be naturally extended to the multiple witnesses case. Intuitively, it states that once the pre-processing phase has been executed (where either the prover or the verifier was honest), then for any statement $x \in \mathcal{L}$, two different sets of efficient prover and verifier strategies cannot produce two different transcripts of the execution phase that will both be accepted by the observer.

We show that the CUT property implies steganography freeness. Further, we note that the observer soundness property is crucial in natural applications of SF-ZK. Indeed, if we use SF-ZK to implement a GMW-style compiler for constructing steganography-free protocols, then observer soundness would be necessary to ensure that an adversarial party cannot deviate from a prescribed strategy in the underlying protocol and therefore cannot use the execution transcript to perform steganographic communication.

We refer the reader to Section 3.1 for a formal definition of SF-ZK.

**Positive Results.** We construct an SF-ZK argument system with black-box simulation for all languages in NP. We, in fact, provide two constructions: first, assuming sub-exponentially hard injective one-way functions, we devise a solution in the *single-execution* setting, where the pre-processing phase can only be used once. Then, assuming the existence of fully homomorphic encryption [27], we present a solution in the *multi-execution* setting, where the pre-processing can be refreshed to allow for an unbounded number of execution phases.

Our construction of SF-ZK directly works for circuit satisfiability and avoids any use of expensive NP reductions. In Section 4, we provide a construction of SF-ZK in the single-execution setting. While this protocol follows a conceptually clean approach, it involves a computationally expensive sub-protocol where the prover is required to give a "proof of proof," namely, proof of honest behavior in the execution of another proof. To obtain a more efficient solution, we also present another construction that follows the same key ideas as in our first construction but avoids the expensive sub-protocol by instead using *cut-and-choose* techniques [42].

In Section 5, we extend our construction of SF-ZK to the multi-execution setting.

**Optimality of our Model.** In Section 7, we show that our adversarial model is "tight". Specifically, we show that when *both* the prover and the verifier are malicious during the pre-processing, SF-ZK is *impossible*, except for languages in BPP.

### 1.2 Applications

In the following we highlight a few interesting applications of SF-ZK.

**Online Games.** Imagine a group of players that want to engage in a game of poker without a trusted dealer. The standard solution for this is to use a multi-party computation (MPC) protocol to simulate a dealer by combining the randomness of all players. MPC is however an inherently randomized machinery and the same randomness could be used by colluding players to communicate information (say, about their hands) in an undetectable way. This problem was considered in [39], where the authors proposed a solution based on generic MPC together with unique ZK proof.[5] Their solution relies on players physically exchanging sealed envelopes prior to the execution of the protocol and hence cannot be used over the internet (see Section 1.4 for a more detailed comparison).

In contrast, using SF-ZK allows us to bypass any physical interaction among participants at the cost of a non-interactive pre-processing phase. The resulting protocol is sanitized from any covert communication, since transmitting information covertly via SF-ZK is computationally hard.

**Private Classifier.** Consider the scenario where a server holds a trained classifier and wants to give clients oracle access to the prediction without revealing the logic implemented by the predictor. At the same time, the client wants to be assured that the answers of the server are consistent and indeed correspond to the output of the classifier. An obvious solution to this problem is to augment the client-server interaction with a standard ZK proof of correctness.

Consider the event the server gets infected by a virus. The malicious program might instruct the machine to simply output the full description of the classifier. However, such behavior is easy to detect for anyone observing the network traffic. What if the virus implements a more clever strategy: use the ZK proof as a vector to slowly exfiltrate secret information? Since ZK proofs must be randomized, there is plenty of room to transmit information in an undetectable manner.

One solution is to use SF-ZK instead: the (computational) uniqueness of the transcripts ensures that the virus cannot embed information in the randomness of the protocol and observer soundness forces the server to behave correctly. That is, whatever the client can learn from an infected machine he can also learn by honest queries to the non-corrupted server. Note that in this scenario we can assume that the server is not infected during the training of the model, which can be paired with the computation of the honest prover pre-processing.

A similar argument applies to any interaction in the client-server setting where the server holds some amount of secret data (e.g., a password file, or, classified emails) and might get infected with a virus.

### 1.3 Our Techniques

In this section, we provide an overview of the main ideas underlying our constructions of SF-ZK, both in the single-execution and multi-execution settings.

**How to Simulate?** We start by describing a key conceptual challenge in constructing SF-ZK. Recall that a black-box simulator works by rewinding the adversarial verifier potentially multiple times. This involves creating multiple protocol transcripts which are necessarily different (for the rewinding to be

[5] In unique ZK only a single valid proof exists for a given statement-witness pair.

"successful"). This seems to be at odds with the computationally unique transcripts (CUT) property of SF-ZK; indeed, since the simulator is also an efficient algorithm, intuitively, it should also not be able to produce multiple transcripts of the execution phase. This presents a catch-22: how can we achieve ZK property without violating the CUT property (or vice-versa)?

Towards resolving this conundrum, recall that the CUT property is required to hold against two *different* pairs of prover and verifier strategies $(P_1, V_1)$ and $(P_2, V_2)$, who cannot communicate with each other. This rules out *oblivious* black-box simulation strategies that involve running multiple execution threads (with a common prefix) in parallel since such a strategy implies multiple transcript choices during an honest execution. However, it does *not* rule out *non-oblivious* black-box simulation strategies. In particular, *a non-oblivious simulator can potentially create a transcript, and then use information learned from that transcript to create another one*. This does not violate the CUT property but opens up an avenue for black-box simulation.

**Starting Approach.** To explain our approach, let us first recall the notion of *delayed-input* witness indistinguishable (WI) proofs, where the statement and the witness is only required for computing the last prover message. Such proofs are known in three rounds with a public-coin verifier based on one-way functions [38]. In particular, a recent work of [33] constructed such proofs for circuit satisfiability[6] based on garbled circuits.

Now consider the following template for SF-ZK: during the pre-processing phase, the prover publishes the first message $\alpha$ of the delayed-input WI and additionally commits to some randomness (say) $r$. The verifier commits in advance to the second (public-coin) message $\beta$ of the WI and additionally publishes a "trapdoor" statement with a (verifiably) unique witness. Both the prover and verifier use a non-interactive commitment scheme with unique decommitment[7] to compute their respective commitments.

At the start of the execution phase, both the prover and the verifier receive the statement $x$ and the prover additionally receives a (unique) witness $w$. The execution phase proceeds as follows:

- The prover first simply sends a commitment $c$ to 0 using randomness $r$.
- Next, the verifier decommits to the second message of WI and additionally reveals the (unique) witness for the trapdoor statement.
- Finally, the prover sends the third message $\gamma$ of the WI proof to prove the statement: "either $x$ is true or I committed to the trapdoor witness in $c$ using randomness $r$ that was committed in the pre-processing".

Let us now see why the above template enables black-box simulation. A simulator can first produce a partial transcript of the execution phase by simply committing to 0 in $c$ and then learn the witness for the trapdoor statement. Now, the simulator can rewind the verifier to the start of the execution phase and generate a new transcript where it commits to the trapdoor witness. It then continues the computation of the second transcript and produces the WI proof using the second branch of the statement. Note that the simulator can use the second branch in the WI because it is now true.

**Challenges with CUT.** In order to achieve the CUT property, we require the delayed-input WI proof to have a *unique* accepting third message $\gamma$ for a fixed partial transcript $(\alpha, \beta)$ and a fixed statement and witness. Towards this, let us briefly recall the construction of [33]. Below, we describe the basic version which achieves soundness one half; the full protocol with negligible soundness error is achieved by parallel repetition of the basic protocol.

- First, the prover computes and sends a garbled circuit for the NP verification circuit. Additionally, it commits to all the wire labels of the garbled circuit.
- Next, the verifier sends a random challenge bit.
- If the challenge bit is 0, the prover "opens" everything by revealing its random tape, otherwise, it decommits to wire labels corresponding to the statement and the witness. In the latter case, the verifier simply evaluates the garbled circuit to check if its output is accepting.

[6] The choice of circuit satisfiability as the language is not arbitrary. We use it to avoid the potential issue of using NP reductions that do not preserve the number of witnesses, which can open up an avenue for subliminal communication.

[7] Such schemes are known based on injective one-way functions.

At a first glance, it may seem that the above construction satisfies the unique third message property if the witness is unique. A closer inspection, however, reveals a subtle problem when we use the above WI in our template for SF-ZK. The issue is that a cheating prover can simply guess in advance, e.g., the first index (among all the parallel repetitions) where the challenge bit is 1. In that repetition, he can choose to garble a trivial circuit that outputs 1 on every input. Clearly, in this case, there are exponentially many accepting third messages. As such, the adversarial prover can violate the CUT property with non-negligible probability.

Towards addressing the above problem, our first observation is that the above protocol can be transformed into one that satisfies the unique third message property *at the cost of losing the delayed input property*. The transformation is simple: for every repetition, the prover pre-commits to both of its possible third messages (one for every challenge bit) in the first round. Now, in the last round, it simply decommits to the appropriate response. Clearly, this protocol satisfies the unique third message property but is no longer delayed input since the prover must know the statement and the witness in order to compute the first message. The latter means that we can not directly use it in our template for SF-ZK.

Nevertheless, as we now describe, the above observation can be used to construct a delayed-input WI with the required property. Our main observation is as follows: the aforementioned attack required the prover to deviate from the honest strategy, namely, sending a garbling of a circuit different from the NP verification circuit (i.e., the circuit which outputs 1 on every input). If we could ensure that the prover garbled the "correct" circuit, then the protocol would indeed satisfy the aforementioned uniqueness property.

Towards this end, we modify the protocol template and now require the prover to additionally prove via a separate three-round proof system that it computed the garbling in the first round message of delayed-input WI "honestly". Crucially, *a non-delayed-input proof with unique third message suffices for this task* since the statement and the witness is known in advance. The first and second messages of this proof are fixed in the pre-processing (in a manner as discussed before in the template); the prover only sends the third message of the proof in the execution phase. The uniqueness of this message ensures that it cannot be used for subliminal communication. More importantly, the soundness of this proof ensures that the prover's first message in the delayed-input WI is well-formed, and therefore, the last message is unique.[8]

**Challenges in ZK.** The above idea resolves the main challenge in achieving CUT property, but creates a new challenge in achieving the ZK property. Specifically, the main issue is that in order to perform simulation, it seems that we need the non-delayed-input proof to itself be a (steganography-free) ZK proof. However, this is very close to the goal we started with in the first place.

To resolve this seeming circularity, we observe that the non-delayed-input proof does *not* always need to be simulated. In particular, this proof would only need to be simulated when we invoke the WI property of the delayed-input WI inside the hybrids for proving the ZK property of our main SF-ZK construction. Therefore, we do not need this proof to satisfy the standard notion of ZK with polynomial-time simulation, and instead, it suffices to use ZK with *super-polynomial-time* simulation. Indeed, the super-polynomial-time simulator would only be invoked in the "intermediate" hybrids, but not the final one; therefore, the running time of our final simulator for SF-ZK is *unaffected*. Fortunately, the three-round proof system we described earlier indeed satisfies the super-polynomial-time simulation property.

**Observer Soundness.** While the above solution template resolves the main challenges in achieving ZK and CUT properties, it does not achieve observer soundness property of SF-ZK. Indeed, consider the scenario where the verifier is malicious during the pre-processing phase and uses some a priori fixed randomness (e.g., all 0's). Now, in the execution phase, a malicious prover can use the trapdoor witness (i.e., the witness of the second branch) in the WI proof in the last round.

To address this challenge, we observe that if the verifier is dishonest during pre-processing, then by our assumption that at least one of the parties be honest, we have that the prover must be honest during pre-processing. We use this observation to create an "asymmetry" between a malicious prover and the simulator. Specifically, we require the prover to commit to bit 0 in the pre-processing phase. We also

[8] We remark that our actual protocol slightly differs from the above description in that instead of using delayed-input WI, we introduce and use the notion of (computationally) unique non-interactive WI with honest prover pre-processing. This approach yields a more simplified construction. In this Section, however, we ignore this distinction.

modify the second branch of the WI in the execution phase. Specifically, the second branch will now additionally require the prover to prove that it committed to 1 in the pre-processing phase. Note that since the prover was honest in the pre-processing, it can never execute the second branch since it is always false. However, a simulator can choose to commit to 1 in the pre-processing phase and therefore still use the second branch of the WI.

**Other Details.** The above discussion is oversimplified and ignores several additional technical issues that we need to address to obtain a secure construction of SF-ZK. For example, we must deal with aborting verifiers who may choose to abort on one of the branches of WI with a high probability to skew the distribution of transcripts generated by the simulator. We also need to enable some mechanism for proving soundness as well as the CUT property via *extraction*, even when the verifier's randomness is fixed during the pre-processing. We resolve these issues by using techniques from [28], and by relying on complexity leveraging in some of our proofs. We refer the reader to the technical Sections for more details.

**Multi-Execution SF-ZK.** The pre-processing phase of the above construction is *non-reusable*, i.e., it can only be used for a single execution phase. We now describe a strategy to *refresh* the pre-processing phase. Our starting idea is simple: During the $i$-th execution phase, the prover and the verifier simply generate new pre-processing messages using pre-committed randomness and give a new SF-ZK proof to establish that the new message was computed honestly. Note, however, that in regular ZK proofs, the size of the prover's message grows with the size of the relation circuit. This means that the size of the $i$-th pre-processing messages must be larger than the size of the $(i+1)$-th pre-processing messages, at least by a multiplicative overhead of the security parameter. This means that this approach becomes infeasible after a constant number of refreshes.

A plausible approach to allow unlimited refreshing is to use an SF-ZK where the communication complexity does not grow with the size of the relation circuit. Four round ZK arguments (without SF property) that satisfy such a succinctness property are known for all of NP based on collision-resistant hash functions [37]. Unfortunately, it is not clear how to use such argument systems in our setting: first, we need the argument system to be *delayed-input*, namely, where the first message of the prover is independent of the statement. Further, it is unclear how to force uniqueness of last prover message while only relying on *non-interactive* pre-processing.

We instead use a different solution based on (leveled) fully-homomorphic encryption. The main idea is that instead of having the prover perform an "expensive" computation and prove its validity to the verifier, we instead require both the prover and the verifier to perform the expensive computation "locally" on their own. Since the computation involves the private state of the prover, we use FHE to send it to the verifier, who can use the homomorphism property to perform the computation. Now, the prover only needs to prove a simple statement that the resulting encryption (after homomorphic evaluation) decrypts to the "correct" value. The size of this statement (and the corresponding relation circuit) is fixed, and does not cause a blowup as before. Also observe that the maximum size of the circuit to be computed homomorphically is a priori fixed, therefore leveled FHE suffices. We note that this idea has been previously used (see, e.g., [35]) to construct "short" non-interactive zero-knowledge proofs.

### 1.4 Related Work

Preventing steganographic communication has been the subject of a large body of literature addressing the problem in variety models. We provide a short summary of other directions that address the challenge of protecting cryptosystems against different forms of subversion in below (also refer the reader to [43] for an excellent comprehensive survey).

**Collusion-Free Protocols.** Our work is closely related to prior work on collusion-free protocols [39] (see also [40]). Roughly speaking, a collusion-free multiparty protocol prevents a group of adversarial parties from colluding with each other to gain an unfair advantage over honest participants, e.g., in a game of poker. As Lepinski *et al.* explain in their work, a key challenge in designing such protocols is preventing steganographic communication between the adversarial parties. They use *physical assumptions*, namely, simultaneous exchange of sealed envelopes, and an *interactive* pre-processing model to construct collusion-free protocols. While their overall goal is very similar to ours, we note that their constructions require strong physical assumptions (e.g., sealed envelopes) to ensure verifiable determinism.

We further note that our notion of steganography-free ZK is similar in spirit to the notion of "unique ZK" [40], which is used by [39] in their constructions. In particular, unique ZK requires a one-to-one mapping between a proof transcript and the witness used to compute the transcript, which is similar to the CUT property of steganography-free ZK. However, while our notion of steganography-free ZK is a *strengthening* of zero-knowledge, the notion of unique ZK is not. Unique ZK requires a common reference string as well as a pre-processing step where the prover must necessarily be honest. This means that if the prover was dishonest from the beginning, the soundness no longer holds (even if the verifier continues to be honest from the beginning). Unique ZK also does not require the observer soundness property, which makes it harder to use in our applications.

**Preventing Steganography via Sanitization.** Multiple lines of works have used the approach of using "sanitization" to prevent steganographic communication. The work of Alwen *et al.* [1] considered a mediator model for collusion-free protocols to avoid the use of pre-processing and physical channels. This active mediator has the ability to modify the messages of the protocol participants. This approach is similar in spirit to prior work on subliminal-free ZK and divertible ZK protocols [18, 44, 11, 13, 9, 12] who also use an active "warden" to modify the messages of the prover and the verifier. More recently, Mironov and Stephens-Davidowitz [43] (see also [21]) initiated the study of "reverse firewalls" to prevent steganographic communication in general two-party communication. Roughly speaking, a reverse firewall for a party $P$ is an external entity that sits between $P$ and the outside world and whose scope is to sanitize $P$s incoming and outgoing messages in the face of subversion of their computer. Later, there has been more efforts on secure computation protocols in this model [15, 26, 16].

*Comparison to our model.* In the sanitization-based model there is an entity (namely, the reverse firewall) that sits on the network of each participant and has the ability to re-randomize the messages sent by the parties. We note that all of these works differ fundamentally from ours in that they rely on an *active* mediator (or warden, or reverse firewall) who can sanitize the messages of the parties, whereas we consider the classical steganographic communication setting, where there is a *passive* observer who can look at the messages of the parties (but not modify them). This allows one to *detect* steganography by just looking at the communication transcript.

**Kleptography and Algorithmic-Substitution Attacks.** A sequence of works starting from [50, 51], and more recently followed by a series of papers [8, 5, 7, 46, 47, 2], consider the problem of designing cryptographic primitives which retain meaningful security even against adversaries who can tamper with the implementation of the cryptographic algorithm. In particular, these works consider "functionality-preserving" tampering where the adversary does not break the functionality of the cryptographic algorithm to avoid detection. However, this still leaves open the possibility of the tampered implementation leaking any secret information used by the cryptographic algorithm (e.g., a secret-key for encryption, or a signing key for signature schemes) to the adversary by misusing the randomness. For this reason, these works either avoid the use of randomness altogether (whenever possible), or rely on external sanitizers (such as random oracles) or consider split-state tampering.

There has been another the line of work for protection mechanisms by Dodis *et al.* [20] that studies backdoored pseudorandom generators (BPRGs). In their setting, public parameters are secretly generated together with secret backdoors by a subversive that allows to bypass security, while for any adversary that does not know the backdoor it remains secure.[9] They showed that BPRGs can be immunized by applying a non-trivial function (e.g., a PRF or a seeded extractor) to the outputs of a possibly backdoored pseudorandom generator.

*Comparison to our model.* Our setting (involving ZK proofs and multi-party computation) necessarily relies on the use of randomness. As such, the solutions we achieve in our model restrict the use of randomness to the pre-processing step, without relying on external sanitizers, or other such means.

**Trusted Initialization Phase.** Assuming the trust initialization phase setting, Fischlin and Mazaheri [25] proposed an alternative defense mechanism, so-called self-guarding that contrary to the aforementioned approaches that rely on external sanitizers, does not depend on external parties. The security definitions in this model rely on the assumption of having a "secure initialization phase". This assumption makes our problem substantially easier: The NIZK by Sahai and Waters [48] has a deterministic

[9] Parameter subversion has been considered for several primitives, including pseudorandom generators [20, 17], non-interactive zero knowledge [4], and public-key encryption [2].

prover and it trivially yields a construction of steganography-free ZK in the common reference string (CRS) model.

*Comparison to our model.* Self-guarding requires one to rely on a trusted initialization phase where the cryptosystem is unsubverted. In our model, each party runs a local pre-processing, and security is guaranteed if *either of the parties* is honest during the pre-processing phase.

## 2 Preliminaries

We denote by $n \in \mathbb{N}$ the security parameter that is implicitly given as input to all algorithms in unary representation $1^n$. We denote by $\{0,1\}^\ell$ the set of all bit-strings of length $\ell$. For a finite set $S$, we denote the action of sampling $x$ uniformly at random from $S$ by $x \leftarrow\!\!\$\, S$, and we denote the cardinality of $S$ by $|S|$. An algorithm is efficient or $\mathsf{PPT}$ if it runs in time polynomial in the security parameter. If $\mathcal{A}$ is randomized then by $y := \mathcal{A}(x;r)$ we denote that $\mathcal{A}$ is run on input $x$ and with random coins $r$ and produces output $y$. If no randomness is specified, then it is assumed that $\mathcal{A}$ is run with freshly sampled uniform random coins, and we write this as $y \leftarrow \mathcal{A}(x)$. A function $\mathsf{negl}(n)$ is negligible if for all positive polynomial $\mathsf{poly}(n)$, there exists an $N \in \mathbb{N}$, such that for all $n > N$, $\mathsf{negl}(n) \leq 1/\mathsf{poly}(n)$.

### 2.1 Projective Garbling

We recall the notion of projective garbling schemes as presented in [6].

**Definition 1 (Garbling Scheme).** *A projective garbling scheme is a pair of algorithms* $(\mathsf{Gb}, \mathsf{Eval})$ *defined as follows.*

- *The garbling algorithm* $\mathsf{Gb}$ *takes as input a circuit* $\mathcal{C}$ *and a randomness* $r$ *and outputs a garbled circuit* $C$ *and a projective encoding* $e := (e_1^0, e_1^1, \ldots, e_n^0, e_n^1)$.
- *The evaluation algorithm* $\mathsf{Eval}$ *takes as input a garbled circuit* $C$ *and an encoding* $(e_1, \ldots, e_n)$ *and returns an output* $y$.

For correctness we require that for all inputs $x \in \{0,1\}^n$, for all random tapes $r$, for all circuits $\mathcal{C}$, and for all $(C, e) \in \mathsf{Gb}(\mathcal{C}, r)$ it holds that $\mathsf{Eval}(C, (e_1^{x_1}, \ldots e_n^{x_n})) = \mathcal{C}(x)$. We require the garbling scheme to be private.

**Definition 2 (Privacy).** *A garbling scheme* $(\mathsf{Gb}, \mathsf{Eval})$ *achieves privacy if there exists a* $\mathsf{PPT}$ *simulator* $\mathsf{Sim}$*, such that for all* $\mathsf{PPT}$ *distinguishers* $\mathcal{D} := (\mathcal{D}_1, \mathcal{D}_2)$

$$\left| \begin{array}{l} \Pr_{r \leftarrow \{0,1\}^n}\left[(\mathcal{C}, x) \leftarrow \mathcal{D}_1, (C, e) \leftarrow \mathsf{Gb}(\mathcal{C}, r) : \mathcal{D}_2(C, e_1^{x_1}, \ldots, e_n^{x_n})\right] \\ - \Pr\left[(\mathcal{C}, x) \leftarrow \mathcal{D}_1 : \mathcal{D}_2(\mathsf{Sim}(\mathcal{C}(x)))\right] \end{array} \right| \leq \mathsf{negl}(n)\,.$$

### 2.2 Homomorphic Encryption

We recall the notion of homomorphic encryption [27].

**Definition 3 (Homomorphic Encryption).** *A homomorphic encryption scheme* $(\mathsf{KGen}, \mathsf{Enc}, \mathsf{Dec}, \mathsf{Eval})$ *for a function family* $\mathcal{F}$ *is composed of the following* $\mathsf{PPT}$ *algorithms.*

- *The key generation algorithm* $\mathsf{KGen}$ *takes as input the security parameter* $1^n$ *and returns a key pair* $(\mathsf{sk}, \mathsf{pk})$.
- *The encryption algorithm* $\mathsf{Enc}$ *takes as input a public key* $\mathsf{pk}$ *and a message* $m$ *and returns a ciphertext* $c$.
- *The evaluation algorithm* $\mathsf{Eval}$ *takes as input a public key* $\mathsf{pk}$*, a ciphertext* $c$ *and a function* $f$ *and returns an evaluated ciphertext* $c$.
- *The decryption algorithm takes as input a secret key* $\mathsf{sk}$ *and a ciphertext* $c$ *and returns a message* $m$.

The homomorphic encryption scheme is correct if for all functions $f \in \mathcal{F}$, all $(\mathsf{sk}, \mathsf{pk}) \in \mathsf{KGen}(1^n)$, all messages $m$, all ciphertexts $c \in \mathsf{Enc}(\mathsf{pk}, m)$, then $\mathsf{Dec}(\mathsf{sk}, \mathsf{Eval}(\mathsf{pk}, c, f)) = f(m)$. If $\mathcal{F}$ is the family of all polynomially computable functions we say that the encryption scheme is *fully* homomorphic (FHE). Furthermore, if the maximum size of the computable circuit is a priori bounded, we say that the FHE is *levelled*. We say that a scheme is secure if it satisfies the standard notion of CPA-security [31].

### 2.3 Zero-Knowledge with Super-Polynomial Simulation

Here we recall the notion of zero-knowledge arguments with super-polynomial simulation and extraction (SPS-ZK) [45].

**Definition 4 (Super-Polynomial Simulation).** *An interactive proof* $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ *for some language* $\mathcal{L}$ *with witness relation* $\mathcal{R}$ *is simulatable in super-polynomial time* $T$ *if for every* $\mathsf{PPT}$ *machine* $\mathsf{V}^*$ *there exists a simulator* $\mathsf{Sim}$*, with running time bounded by* $T$*, such that the following ensembles are statistically indistinguishable.*

$$\{\langle \mathsf{SPS\text{-}P}(w,x), \mathsf{V}^*(x)\rangle\}_{x\in\mathcal{L}} \approx \{\mathsf{Sim}(x)\}_{x\in\mathcal{L}}$$

**Definition 5 (Super-Polynomial Extraction).** *An interactive proof* $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ *for some language* $\mathcal{L}$ *with witness relation* $\mathcal{R}$ *is extractable in super-polynomial time* $T$ *if for every* $\mathsf{PPT}$ *machine* $\mathsf{P}^*$ *that convinces the verifier with non-negligible probability on some* $x$*, there exists an extractor oracle machine* $\mathsf{Ext}$*, with running time bounded by* $T$*, such that* $(\mathsf{Ext}^{\mathsf{P}^*}(x), x) \in \mathcal{R}$ *with all but negligible probability.*

We assume the existence of a three-round, public-coin zero-knowledge protocol $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ for NP with the following properties:

1. The protocol is simulatable in super-polynomial time.
2. For any first and second round message pair $(\alpha, \beta)$, there exists only one accepting third message $\gamma$.
3. Fix an accepting transcript $(\alpha, \beta, \gamma)$, then either of the following conditions must hold:
   (a) The witness is extractable (possibly in super-polynomial time) from $(\alpha, \beta, \gamma)$.
   (b) There is at most one $\beta$ such that $(\alpha, \beta, \gamma)$ is accepting.

Many three-round protocols can be shown (or easily adapted) to satisfy these properties, e.g., the three-round proof for graph Hamiltonicity [24]. In the following we describe a simple and efficient protocol based on Yao's garbled circuits [49, 6] and cut-and-choose: The prover garbles $s$ independent copies of the verification circuit with the statement hardwired. Then, for each garbled circuit $C^{(i)}$, it computes a (perfectly-binding) commitment $c_0^{(i)}$ to the random coins used to garble $C^{(i)}$ and a (perfectly-binding) commitment $c_1^{(i)}$ to the labels corresponding to the witness $w$. The first message $\alpha$ corresponds to the collection of the circuits $(C^1, \ldots, C^{(s)})$ together with the corresponding commitments. The individual bits of the challenge $\beta \in \{0,1\}^s$ determine whether the prover should reveal the coins or the labels for the $i$-th circuit and the message $\gamma$ contains the openings of the corresponding commitments. The verifier accepts if the circuits for which the coins are opened (check circuits) compute the correct relation and all of the others (evaluation circuits) output 1.

The protocol is simulatable in super-polynomial time (in the sense of [45]) by guessing the challenge $\beta$ ahead of time. Also, fixing $\alpha$ and $\beta$, the last message $\gamma$ is clearly unique since the commitment scheme is perfectly binding and has unique openings. To show that the third condition is satisfied, assume that the witness is not extractable from a valid transcript $(\alpha, \beta, \gamma)$. Then for all check circuits, it must hold that $c_1^{(i)}$ does not contain a valid set of labels, as otherwise, one could simply read the witness using the revealed random coins. On the other hand, for all evaluation circuits, $c_0^{(i)}$ cannot contain a set of coins that lead to a correct garbling of the relation as otherwise the witness would be uniquely determined by the labels (known in plain). This however means that the set of commitments $(c_0^{(1)}, c_1^{(1)}, \ldots, c_0^{(s)}, c_1^{(s)})$ uniquely determines the challenge $\beta$.

### 2.4 NIWI with Honest Pre-Processing

Non-Interactive Witness Indistinguishable Arguments with Honest Pre-Processing (HPP-NIWI) are a special kind of non-interactive witness indistinguishable arguments where computation of the argument is split into two steps. In an initial *statement-independent* step the prover commits to a preliminary value $\tau$. In the second step, which now depends on the statement $x$ to be proven, the actual argument $\pi$ is computed. The soundness of an HPP-NIWI is only guaranteed as long as the pre-processing step is performed honestly, in the sense that there must exist some random coins such that the corresponding execution of the honest prover results in $\tau$.

This kind of argument is very useful for our purposes because we will require that all messages sent in the execution phase of our protocols are computationally unique. If we were to use regular NIWI in such a protocol, we could never hope to achieve this, since an argument with a deterministic prover cannot be witness indistinguishable. However, an HPP-NIWI allows us to segregate all of the randomnesses of the prover into the first, statement-independent step, such that for any honest preliminary value $\tau$ and any fixed statement $x$ the prover can produce only a single unique valid argument $\pi$. We formally define HPP-NIWIs in the following.

**Definition 6 (Non-Interactive Witness Indistinguishable Arguments with Honest Pre-processing).** *A non-interactive witness indistinguishable argument with honest pre-processing (HPP-NIWI) for a language $\mathcal{L}$ with corresponding relation $\mathcal{R}$ is a triple of algorithms $\mathsf{NIWI} = (\mathsf{WI\text{-}P}_1, \mathsf{WI\text{-}P}_2, \mathsf{WI\text{-}V})$ such that the following holds.*

*Correctness. For all $(x, w) \in \mathcal{R}$, all randomness $r \in \{0,1\}^n$, all $\tau \leftarrow \mathsf{WI\text{-}P}_1(r)$, and all $\pi \leftarrow \mathsf{WI\text{-}P}_2(x, w, r)$ it holds that $\mathsf{WI\text{-}V}(x, \tau, \pi) = 1$.*

*Soundness. For all malicious provers $\mathsf{WI\text{-}P}^*$ there exists a negligible function such that*

$$\Pr\left[(x^*, \tau^*, \pi^*, r^*) \leftarrow \mathsf{WI\text{-}P}^*(1^n) : \begin{matrix} x^* \notin \mathcal{L} \wedge \mathsf{WI\text{-}P}_1(r^*) = \tau^* \\ \wedge \ \ \mathsf{WI\text{-}V}(x^*, \tau^*, \pi^*) = 1 \end{matrix}\right] \leq \mathsf{negl}(n).$$

*Witness Indistiguishability. For any $(x, w_1, w_2)$ such that $(x, w_1) \in \mathcal{R}$ and $(x, w_2) \in \mathcal{R}$, all randomness $r \in \{0,1\}^n$, and $\tau \leftarrow \mathsf{WI\text{-}P}_1(r)$ it holds that for all PPT algorithms $\mathsf{D}$, there exists a negligible function such that*

$$\left|\begin{matrix} \Pr\left[\pi_1 \leftarrow \mathsf{WI\text{-}P}_2(x, w_1, r) : \mathsf{D}(\tau, \pi_1) = 1\right] \\ -\Pr\left[\pi_2 \leftarrow \mathsf{WI\text{-}P}_2(x, w_2, r) : \mathsf{D}(\tau, \pi_2) = 1\right] \end{matrix}\right| \leq \mathsf{negl}(n).$$

*Extractability. There exists a PPT extractor $\mathsf{Ext}$, such that for any $x^* \in \mathcal{L}$, any randomness $r \in \{0,1\}^n$, and $\tau^* \leftarrow \mathsf{WI\text{-}P}_1(r)$ and all $\pi^*$ such that $\mathsf{WI\text{-}V}(x^*, \tau^*, \pi^*) = 1$ there exists a negligible function such that*

$$\Pr\left[w^* \leftarrow \mathsf{Ext}(x^*, \tau^*, \pi^*, r) : (x^*, w^*) \in \mathcal{R}\right] \geq 1 - \mathsf{negl}(n).$$

*Note that this does not contradict witness indistinguishability, since the extractor receives the prover's secret randomness as input.*

An HPP-NIWI is said to have unique proofs, if for each $\tau \leftarrow \mathsf{WI\text{-}P}_1(r)$ and each $(x, w) \in \mathcal{R}$ there exists at most one $\pi$ such that $\mathsf{WI\text{-}V}(x, \tau, \pi) = 1$. See Appendix A for more details of the HPP-NIWI construction.

## 3 Defining Steganography-Freeness

In this section, we introduce the definitions of steganography-free zero-knowledge interactive arguments and steganography-free multi-party computation. Steganography-freeness is generally impossible for regular protocols because without being constrained, a malicious party could always try to correlate its randomness with the secrets it wishes to subliminally communicate. We prevent such attacks by utilizing a non-interactive pre-processing phase. Specifically, we consider protocols that proceed in two phases: A non-interactive pre-processing phase, and an interactive execution phase. As we will see below, our definitions guarantee that no steganographic communication can be performed in the execution phase, once the pre-processing was completed.

We begin by defining steganography-freeness for generic interactive protocols (with pre-processing), which closely matches the intuition behind this notion. Roughly speaking, our notion steganography-free says that no machines can communicate through a protocol execution without being detected. This is captured as a game between a sender and a receiver, where the sender is given a random bit $b$ and interacts with the receiver. In order to win the game the receiver must output $b$, without raising the suspicion of an external observer. The formal definition is given in the following.

**Definition 7 (Steganography-Freeness).** *A protocol $\Pi = (\mathsf{S}^1, \mathsf{R}^1, \mathsf{S}^2, \mathsf{R}^2)$ is steganography-free relative to a PPT observer $\Theta$ if for all admissible pairs $(\tilde{\mathsf{S}}^1, \tilde{\mathsf{R}}^1)$, and for all PPT algorithms $(\mathsf{S}^*, \mathsf{R}^*)$ it holds that*

$$\Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \tilde{\mathsf{S}}^1(1^\lambda), (s_2, p_2) \leftarrow \tilde{\mathsf{R}}^1(1^\lambda), \\ b \leftarrow_\$ \{0,1\}, \mathcal{T} := \langle \mathsf{S}^*(s_1, p_2, b), \mathsf{R}^*(s_2, p_1)\rangle\end{matrix} : \begin{matrix}\Theta(p_1, p_2, \mathcal{T}) = 1 \wedge \\ \mathsf{R}^*(s_2, p_1, \mathcal{T}) = b\end{matrix}\right] \leq \frac{1}{2} + \mathsf{negl}(n)$$

*where* $(\mathsf{S}^*, \mathsf{R}^*)$ *are the (possibly) corrupted versions of* $(\mathsf{S}^2, \mathsf{R}^2)$*. Both parties (*$\mathsf{S}^1$ *and* $\mathsf{R}^1$*) individually compute pre-processing information comprising of a public output and a secret state in the pre-processing stage. In the execution phase, both parties (*$\mathsf{S}^2$ *and* $\mathsf{R}^2$*) receive as input their respective secret states as well as the other party's public output from the pre-processing phase.*

Note that the definition is *relative* to some observer $\Theta$. Generally, any protocol is steganography-free relative to *some* observer, e.g., the trivial $\Theta$ that does not accept *any* transcript. However, this is of course not a useful property. The challenge, therefore, is to achieve steganography-freeness relative to a meaningful observer that accepts honest communication.

It is also important to observe that the definition is conditioned on some admissibility criterion on the behavior of the players in the pre-processing. In this work we are interested in what we call a *partial-honest* pre-processing, i.e., a pair $(\tilde{\mathsf{S}}^1, \tilde{\mathsf{R}}^1)$ is considered admissible if both algorithms are $\mathsf{PPT}$ and at least one of them is honest. Note that for this case we consider *rushing* adversaries that sample their pre-processing after the honest one is fixed. We mention that the definition can be extended to capture a bounded amount of covert communication by sampling multiple bits.

### 3.1 Steganography-Free Zero-Knowledge

Towards defining steganography-free zero-knowledge, we extend the standard definitions in a natural way to accommodate an input-independent pre-processing phase. In the pre-processing stage, both parties ($\mathsf{P}^1$ and $\mathsf{V}^1$) individually compute pre-processing information comprising of a public output and a secret state. In the execution phase, both parties ($\mathsf{P}^2$ and $\mathsf{V}^2$) receive as input their respective secret states as well as the other party's public output from the pre-processing phase, together with the statement $x$. The prover additionally receives a witness $w$. At the end of this phase, the honest verifier outputs either 0 or 1. In addition to the standard properties for a zero-knowledge protocol, a steganography-free zero-knowledge protocol must additionally satisfy the following new properties:

1. *Observer Completeness:* There exists an efficient algorithm $\Theta$, that takes as input the protocol transcript and accepts if both parties are honest.
2. *Observer Soundness:* The (possibly colluding) prover and verifier cannot convince the observer to accept a transcript for any $x \notin \mathcal{L}$, as long as either the prover or the verifier executes the pre-processing phase honestly.
3. *Computationally Unique Transcripts:* Given a language with unique witnesses, no two independent coalitions of prover and verifier can produce two different transcripts that are both accepted by the observer. This is again conditioned on the fact that at least one of the parties was honest during the pre-processing.

This set of properties will guarantee that the protocol execution cannot be used as a covert channel. Later we will show that these conditions are indeed sufficient to achieve steganography-freeness. The formal definition is given in the following.

**Definition 8 (Steganography-Free Zero-Knowledge Arguments).** *Let* $\mathcal{L}$ *be a language in NP with corresponding relation* $\mathcal{R}$*. A steganography-free interactive argument system* $\Pi = (\mathsf{P}, \mathsf{V})$ *for language* $\mathcal{L}$ *in the non-interactive pre-processing model with observer* $\Theta$ *must satisfy the following properties:*

*Completeness. For all* $(x, w) \in \mathcal{R}$ *it holds that*

$$\Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \mathsf{P}^1(1^n), \\ (s_2, p_2) \leftarrow \mathsf{V}^1(1^n)\end{matrix} : 1 \leftarrow \left\langle \mathsf{P}^2(x, w, s_1, p_2), \mathsf{V}^2(x, s_2, p_1)\right\rangle\right] \geq 1 - \mathsf{negl}\,(n)\,.$$

*Computational Non-Adaptive Soundness. For all* $x \notin \mathcal{L}$ *and all malicious* $\mathsf{PPT}$ *provers* $\mathsf{P}^*$ *it holds that*

$$\Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \mathsf{P}^*(x), \\ (s_2, p_2) \leftarrow \mathsf{V}^1(1^n)\end{matrix} : 1 \leftarrow \left\langle \mathsf{P}^*(x, s_1, p_2), \mathsf{V}^2(x, s_2, p_1)\right\rangle\right] \leq \mathsf{negl}\,(n)\,.$$

*Computational Soundness. For all malicious* $\mathsf{PPT}$ *provers* $\mathsf{P}^*$ *it holds that*

$$\Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \mathsf{P}^*(1^n), \\ (s_2, p_2) \leftarrow \mathsf{V}^1(1^n), \\ x \leftarrow \mathsf{P}^*(s_1, p_2)\end{matrix} : 1 \leftarrow \left\langle \mathsf{P}^*(x, s_1, p_2), \mathsf{V}^2(x, s_2, p_1)\right\rangle \wedge x \notin \mathcal{L}\right] \leq \mathsf{negl}\,(n)\,.$$

*Here, we use the terms computational soundness and adaptive computational soundness interchangeably.*

*Zero-Knowledge. For all malicious* PPT *verifiers* $\mathsf{V}^*$ *there exists an expected polynomial time simulator* $\mathsf{Sim}$*, such that for all* PPT *distinguishers* $\mathcal{D}$*, it holds that for all tuples* $(x, w) \in \mathcal{R}$

$$\left| \begin{array}{l} \Pr\left[ \begin{matrix} (s_1, p_1) \leftarrow \mathsf{P}^1(1^n), \\ (s_2, p_2) \leftarrow \mathsf{V}^*(x) \end{matrix} : \mathcal{D}(\langle \mathsf{P}^2(x, w, s_1, p_2), \mathsf{V}^*(x, s_2, p_1) \rangle) = 1 \right] \\ - \Pr\left[ \begin{matrix} (s_1, p_1) \leftarrow \mathsf{Sim}(1^n), \\ (s_2, p_2) \leftarrow \mathsf{V}^*(x) \end{matrix} : \mathcal{D}(\langle \mathsf{Sim}(x, s_1, p_2), \mathsf{V}^*(x, s_2, p_1) \rangle) = 1 \right] \end{array} \right| \leq \mathsf{negl}(n) .$$

*Observer Completeness. For all* $(x, w) \in \mathcal{R}$ *it holds that*

$$\Pr\left[ \begin{matrix} (s_1, p_1) \leftarrow \mathsf{P}^1(1^n), (s_2, p_2) \leftarrow \mathsf{V}^1(1^n), \\ \mathcal{T} := \langle \mathsf{P}^2(s_1, p_2, x, w), \mathsf{V}^2(s_2, p_1, x) \rangle \end{matrix} : \Theta(p_1, p_2, \mathcal{T}, x) = 1 \right] \geq 1 - \mathsf{negl}(n) .$$

*Non-Adaptive Observer Soundness. For all* $x \notin \mathcal{L}$*, for all admissible pairs* $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$*, for all* PPT *algorithms* $\mathsf{P}^*$ *and* $\mathsf{V}^*$ *it holds that*

$$\Pr\left[ \begin{matrix} (s_1, p_1) \leftarrow \tilde{\mathsf{P}}^1(x), (s_2, p_2) \leftarrow \tilde{\mathsf{V}}^1(x), \\ \mathcal{T} := \langle \mathsf{P}^*(s_1, p_2, x), \mathsf{V}^*(s_2, p_1, x) \rangle \end{matrix} : \Theta(p_1, p_2, \mathcal{T}, x) = 1 \right] \leq \mathsf{negl}(n)$$

*Observer Soundness. For all admissible pairs* $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$*, for all* PPT *algorithms* $\mathsf{P}^*$ *and* $\mathsf{V}^*$ *it holds that*

$$\Pr\left[ \begin{matrix} (s_1, p_1) \leftarrow \tilde{\mathsf{P}}^1(1^n), (s_2, p_2) \leftarrow \tilde{\mathsf{V}}^1(1^n), \\ x \leftarrow \mathsf{P}^*(s_1, p_2); \mathcal{T} := \langle \mathsf{P}^*(s_1, p_2, x), \mathsf{V}^*(s_2, p_1, x) \rangle \end{matrix} : \begin{matrix} \Theta(p_1, p_2, \mathcal{T}, x) = 1 \\ \wedge \ x \notin \mathcal{L} \end{matrix} \right] \leq \mathsf{negl}(n)$$

*where a pair* $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$ *is considered admissible if both algorithms are* PPT *and it holds that* $\tilde{\mathsf{P}}^1(w, x) = \mathsf{P}^1(1^n)$ *or* $\tilde{\mathsf{V}}^1(x) = \mathsf{V}^1(1^n)$*. Notice that, we use the terms observer soundness and adaptive observer soundness interchangeably.*

*Computationally Unique Transcripts. For all* $x \in \mathcal{L}$ *such that there exists a unique* $w$ *such that* $\mathcal{R}(x, w) = 1$*, for all admissible pairs* $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$*, for all* PPT *algorithms* $(\mathsf{P}^*, \mathsf{V}^*, \hat{\mathsf{P}}^*, \hat{\mathsf{V}}^*)$ *it holds that*

$$\Pr\left[ \begin{matrix} (s_1, p_1) \leftarrow \tilde{\mathsf{P}}^1(w, x), (s_2, p_2) \leftarrow \tilde{\mathsf{V}}^1(x), \\ \mathcal{T}_1 := \langle \mathsf{P}^*(s_1, p_2, x, w), \mathsf{V}^*(s_2, p_1, x) \rangle, \\ \mathcal{T}_2 := \left\langle \hat{\mathsf{P}}^*(s_1, p_2, x, w), \hat{\mathsf{V}}^*(s_2, p_1, x) \right\rangle \end{matrix} : \begin{matrix} \Theta(p_1, p_2, \mathcal{T}_1, x) = 1 \wedge \\ \Theta(p_1, p_2, \mathcal{T}_2, x) = 1 \wedge \\ \mathcal{T}_1 \neq \mathcal{T}_2 \end{matrix} \right] \leq \mathsf{negl}(n)$$

*where a pair* $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$ *is considered admissible if both algorithms are* PPT *and it holds that* $\tilde{\mathsf{P}}^1(w, x) = \mathsf{P}^1(1^n)$ *or* $\tilde{\mathsf{V}}^1(x) = \mathsf{V}^1(1^n)$*.*

Observe that, although the honest pre-processing algorithms do not require the statement or the witness as input, we still provide the (possibly) malicious machines with $x$ (and $w$ if the prover is malicious). This guarantees that the properties are preserved even if the algorithm has partial knowledge of the statement (and possibly the witness) ahead of time.

We further remark that our definition of computationally unique transcripts is going to be useful only for languages with unique witnesses, since the prover might be able to produce two accepting transcripts by simply executing the protocol with two different witnesses. While this suffices for our applications, the definition can be naturally extended to the $k$-witnesses case by requiring the coalitions to output $k + 1$ distinct valid transcripts.

**Steganography-Freeness.** In the following, we argue that our conditions defined above suffice to show that the protocol satisfies steganography-freeness.

**Theorem 1 (Steganography-Freeness).** *Let* $\mathcal{L}$ *be a language with unique witnesses and let* $(\mathsf{P}, \mathsf{V})$ *be an observer sound zero-knowledge protocol for* $\mathcal{L}$ *with computationally unique transcripts. Then* $(\mathsf{P}, \mathsf{V})$ *is steganography-free relative to the observer with partially honest pre-processing.*

*Proof.* We assume that the prover plays the role of the sender and the verifier plays the role of the receiver. The other case follows along the same lines. Then we distinguish two cases.

1. $x \notin \mathcal{L}$ : In this case the observer returns 1 with negligible probability and therefore the condition trivially holds.
2. $x \in \mathcal{L}$ : Let $\mathsf{success}$ be the event that the condition of the SF experiments is satisfied. By the law of total probabilities we have that

$$\begin{aligned}\Pr[\mathsf{success}] =& \Pr[\mathsf{success}|\Theta(p_1,p_2,\mathcal{T})=1]\cdot\Pr[\Theta(p_1,p_2,\mathcal{T})=1]\\ &+\Pr[\mathsf{success}|\Theta(p_1,p_2,\mathcal{T})=0]\cdot\Pr[\Theta(p_1,p_2,\mathcal{T})=0]\\ =&\Pr[\mathsf{success}|\Theta(p_1,p_2,\mathcal{T})=1]\cdot\Pr[\Theta(p_1,p_2,\mathcal{T})=1]\end{aligned}$$

since, by definition of $\mathsf{success}$, we have that

$$\Pr[\mathsf{success}|\Theta(p_1,p_2,\mathcal{T})=0]=0.$$

By a trivial bound we have that

$$\begin{aligned}&\Pr[\mathsf{success}] \le \Pr[\mathsf{success}|\Theta(p_1,p_2,\mathcal{T})=1]\\ &=\Pr\left[\begin{array}{l}(s_1,p_1)\leftarrow\tilde{\mathsf{P}}^1(1^\lambda),\\(s_2,p_2)\leftarrow\tilde{\mathsf{V}}^1(1^\lambda),\\b\leftarrow_\$\{0,1\},\\ \underbrace{\langle\mathsf{P}^*(s_1,p_2,b),\mathsf{V}^*(s_2,p_1)\rangle}_{\mathcal{T}}\end{array} : \begin{array}{l}\Theta(p_1,p_2,\mathcal{T})=1\wedge\\ \mathsf{V}^*(s_2,p_1,\mathcal{T})=b\end{array}\middle|\Theta(p_1,p_2,\mathcal{T})=1\right]\\ &\le\Pr\left[\begin{array}{l}(s_1,p_1)\leftarrow\tilde{\mathsf{P}}^1(1^\lambda),\\(s_2,p_2)\leftarrow\tilde{\mathsf{V}}^1(1^\lambda),\\b\leftarrow_\$\{0,1\},\tilde{b}\leftarrow_\$\{0,1\},\\ \underbrace{\left\langle\mathsf{P}^*(s_1,p_2,\tilde{b}),\mathsf{V}^*(s_2,p_1)\right\rangle}_{\mathcal{T}}\end{array} : \begin{array}{l}\Theta(p_1,p_2,\mathcal{T})=1\wedge\\ \mathsf{V}^*(s_2,p_1,\mathcal{T})=b\end{array}\middle|\Theta(p_1,p_2,\mathcal{T})=1\right]\\ &\qquad\qquad+\mathsf{negl}(n)\end{aligned}$$

by the uniqueness of the transcripts, since the transcript $\mathcal{T}$ is identical in both cases with all but negligible probability. Note that in the second game the bit $b$ is information-theoretically hidden and therefore $\mathsf{V}^*$ cannot do better than guessing. Thus we have that

$$\Pr[\mathsf{success}] \le \frac{1}{2}+\mathsf{negl}(n).$$

**Multi-Execution SF-ZK.** The above definition refers to *single-execution* SF-ZK where all of the properties are required to hold for a single execution phase, after the pre-processing is fixed. In Section 5, we extend the notion of SF-ZK to the *multi-execution* setting.

## 4 A Steganography-Free ZK Protocol

Let $\tilde{\mathcal{L}}$ be any average-case hard language with unique witnesses and let $f:\{0,1\}^{n_{\mathsf{OWF}}}\to\{0,1\}^{m_{\mathsf{OWF}}}$ be a one-way function with an efficiently checkable range. Let $(\mathsf{WI\text{-}P},\mathsf{WI\text{-}V})$ be an HPP-NIWI with unique proofs for the following language: $\mathcal{L}_{\mathsf{NIWI}} =$

$$\left\{\begin{pmatrix}x,y,\tilde{w},\\c_0,\bar{c},\tilde{c}\end{pmatrix}\;\middle|\;\begin{array}{l}\exists(w,s,\tilde{r}):((x,w)\in\mathcal{R}\wedge\mathsf{Com}(\tilde{r};s)=\bar{c}\wedge\mathsf{Com}(0^n;\tilde{r})=\tilde{c})\\ \vee\ \exists(r,\tilde{r}):(\mathsf{Com}(1;r)=c_0\wedge\mathsf{Com}(\tilde{w};\tilde{r})=\tilde{c})\\ \vee\ \exists(w,r,z):((x,w)\in\mathcal{R}\wedge\mathsf{Com}(1;r)=c_0\wedge f(z)=y)\end{array}\right\}$$

where the first branch (1) is going to be used by the prover and the second branch (2) will allow one to simulate without knowing the witness. Interestingly the third branch (3) is used neither by the honest prover nor by the simulator, but it is only instrumental to prove the indistinguishability of the two. Finally, we let $(\mathsf{SPS\text{-}P},\mathsf{SPS\text{-}V})$ be a three-round SPS-ZK argument system with unique last messages for the following language:

$$\mathcal{L}_{\mathsf{SPSZK}}=\{\tau\mid\exists u:\mathsf{WI\text{-}P}_1(u)=\tau\}.$$

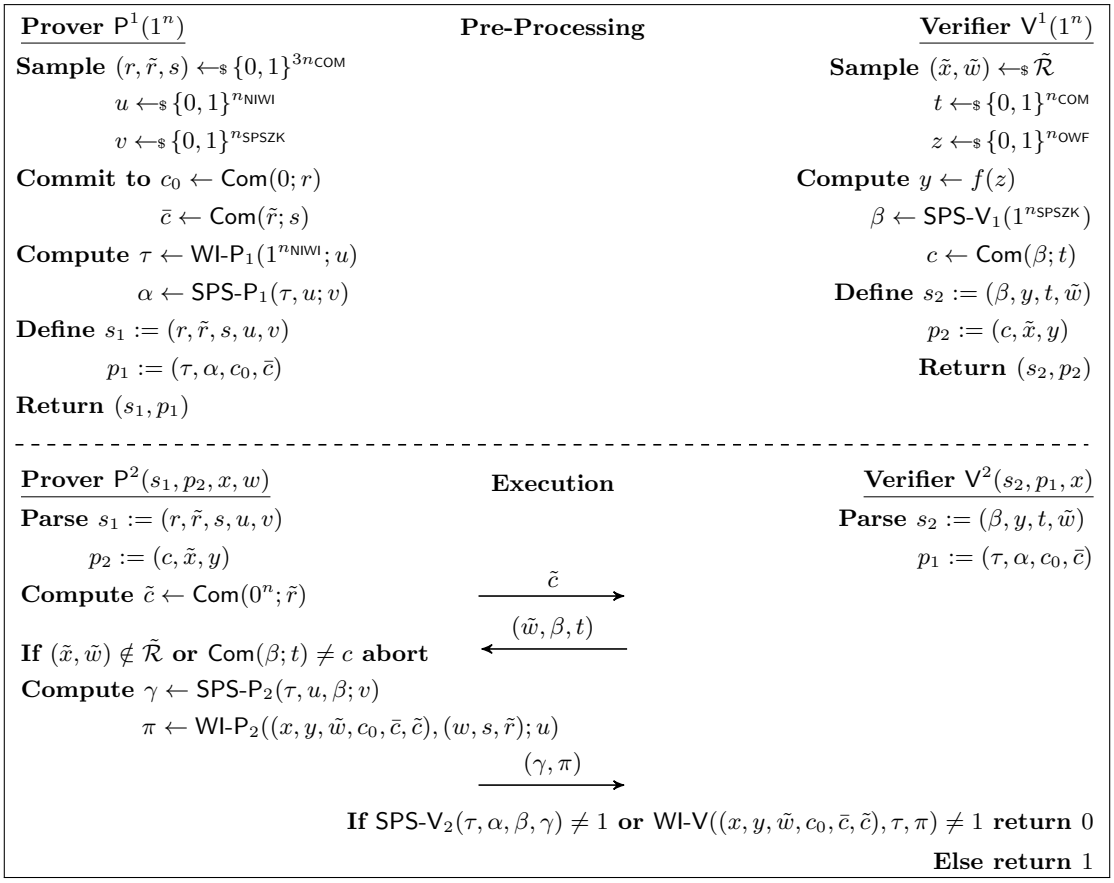


**Fig. 1.** Our SF-ZK protocol.

### 4.1 Our Protocol

Our protocol SF-ZK is formally described in Figure 1. We describe extensions to the multi-execution setting in Section 5.

**Pre-Processing.** In the pre-processing phase, the honest prover computes a commitment to 0 and to some random coins $\tilde{r}$. The former guarantees that, if the prover's pre-processing is honest, then it is hard to cheat in the execution phase, whereas the latter fixes the random coins used later in the execution phase. The prover also initializes the pre-processing $\tau$ of an HPP-NIWI proof and computes the first message $\alpha$ of an SPS-ZK proof that asserts that $\tau$ is well-formed. The public output of the prover's pre-processing consists of the commitments together with the messages $(\tau, \alpha)$. The secret state consists of the random coins used in the pre-processing.

On the other hand, the verifier samples a random image $y$ from the domain of the one-way function $f$ and computes a commitment $c$ to a randomly sampled second message $\beta$ of the SPS-ZK proof. Furthermore, it samples a random instance $\tilde{x}$ of an average-case hard language with unique witnesses. The public output of the verifier's pre-processing consists of $(c, \tilde{x}, y)$, and the secret state consists of the random coins used in the pre-processing.

**Execution.** The execution phase is started by the prover, who sends a commitment $\tilde{c}$ to $0^n$, using the random coins $\tilde{r}$ fixed in the pre-processing. Then the verifier replies with the decommitment $(\beta, t)$ to $c$ and reveals the unique witness $\tilde{w}$. The prover checks that $(\beta, t)$ is a valid decommitment for $c$ and computes the last message $\gamma$ of the SPS-ZK protocol that certifies that $\tau$ is well-formed. Finally, it computes the proof $\pi$ using the first branch (1) thereby proving that $\tilde{c}$ was correctly formed using the random coins committed in the pre-processing and that $x$ is indeed an accepting instance of $\mathcal{L}$. The verifier simply checks whether the transcript $(\alpha, \beta, \gamma)$ and the proof $\pi$ verify correctly.

While $\tilde{c}$ might seem purposeless, it is going to be useful in the simulation: The simulator will spawn a lookahead thread to learn $\tilde{w}$, which will allow it to rewind the execution to compute $\tilde{c}$ as a commitment

to $\tilde{w}$. This in turn allows it to compute the proof $\pi$ using the second branch (2), which does not require knowledge of the witness for $x$. This is however not a feasible strategy for any malicious prover (which cannot rewind the execution of the protocol), since it requires to know $\tilde{w}$ ahead of time.

### 4.2 Analysis

**Parameters.** Let $n$ be the security parameter of our scheme, we consider the following parameters that are (implicitly) given as input to each algorithm of our building blocks:

- $n_{\mathsf{SPSZK}}$ : The security parameter for the SPS-ZK argument $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$.
- $n_{\mathsf{NIWI}}$ : The security parameter for the non-interactive witness indistinguishable argument $(\mathsf{WI\text{-}P}, \mathsf{WI\text{-}V})$.
- $n_{\mathsf{COM}}$ : The security parameter for the perfectly binding commitment scheme $\mathsf{Com}$ with unique openings.
- $n_{\mathsf{L}}$ : The security parameter for the average-case hard language with unique witnesses $\tilde{\mathcal{L}}$.
- $n_{\mathsf{OWF}}$ : The security parameter for the one-way function $f$.

We require that the parameters satisfy the following relation

$$2^{n_{\mathsf{SPSZK}}} \ll 2^{n_{\mathsf{OWF}}} \ll 2^{n_{\mathsf{COM}}} \ll 2^{n_{\mathsf{NIWI}}} = 2^{n_{\mathsf{L}}},$$

where $a \ll b$ means that for all polynomial functions $a \cdot \mathsf{poly}(n) < b$. In particular we require the SPS-ZK argument to be sound against an adversary that runs in time $\mathsf{poly}(n_{\mathsf{SPSZK}})$ and to be simulatable in time $O(2^{n_{\mathsf{SPSZK}}})$. By setting the security parameter of the underlying perfectly binding commitment scheme to be also $n_{\mathsf{SPSZK}}$, then one can find the committed message in time $O(2^{n_{\mathsf{SPSZK}}})$ by exhaustive search.[10] We require the one-way function to be hard to invert in time $O(2^{n_{\mathsf{SPSZK}}})$ but easy to invert in time $O(2^{n_{\mathsf{OWF}}})$, similarly the commitment scheme is hiding against $O(2^{n_{\mathsf{OWF}}})$ bounded machines but extractable in time $O(2^{n_{\mathsf{COM}}})$. Finally, the HPP-NIWI and the average-case hard language shall be hard even for adversaries running in time $O(2^{n_{\mathsf{COM}}}) \gg O(2^{n_{\mathsf{OWF}}}) \gg O(2^{n_{\mathsf{SPSZK}}})$.

**Security proof.** In the following, we state our main theorems:

**Theorem 2 (Soundness).** *If* $(\mathsf{WI\text{-}P}, \mathsf{WI\text{-}V})$ *is an HPP-NIWIs with unique proofs,* $\tilde{\mathcal{L}}$ *is an average-case hard language with unique witnesses,* $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ *is an SPS-ZK argument, and the commitment scheme* $\mathsf{Com}$ *is perfectly binding, then the argument system SF-ZK in Figure 1 is computationally sound.*

*Proof.* The proof consists of two steps. In the first step, we prove that it in present of non-adaptive (selective) security notation in a way that the adversary is not allow to adaptively choose the statement. In the second step, we invoke complexity leveraging to lift the reduction to the adaptive settings.

**Non-adaptive soundness.** Assume that there exists an $x^* \notin \mathcal{L}$ and a malicious $\mathsf{PPT}$ prover $\mathsf{P}^*$ such that the verifier on input $x^*$ and interaction with $\mathsf{P}^*$ will accept with probability $\epsilon$. Let $x_{\mathsf{NIWI}} := (x, y, \tilde{w}, c_0, \bar{c}, \tilde{c})$ We can split this probability into two parts: Either $\mathsf{P}^*$ cheats in such away that $x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}}$ (in which case we will be able to use the soundness of the HPP-NIWI to show that $\mathsf{P}^*$ would not be successful) or $\mathsf{P}^*$ cheats in such away that $x_{\mathsf{NIWI}} \in \mathcal{L}_{\mathsf{NIWI}}$. In this case, we show that this event can only occur with negligible probability due to the average-case hardness of $\tilde{\mathcal{L}}$. Let $\mathsf{cheat}$ be the event that a malicious prover causes the honest verifier to accept $x^*$.

$$\begin{aligned}
\epsilon &= \Pr[\mathsf{cheat}] \\
&= \underbrace{\Pr[\mathsf{cheat} | x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}}] \cdot \Pr[x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}}]}_{\epsilon'} \\
&+ \underbrace{\Pr[\mathsf{cheat} | x_{\mathsf{NIWI}} \in \mathcal{L}_{\mathsf{NIWI}}] \cdot \Pr[x_{\mathsf{NIWI}} \in \mathcal{L}_{\mathsf{NIWI}}]}_{\epsilon''}
\end{aligned}$$

[10] This instantiation of the perfectly binding commitment scheme used inside the SPS-ZK protocol is different from the perfectly binding commitment scheme $\mathsf{Com}$ used in our protocol. In particular, we use different security levels for these schemes.

**Bounding $\epsilon'$.** We will first bound $\epsilon'$ using the soundness of the HPP-NIWI and the super-polynomial extractability of the SPS-ZK. Assume towards contradiction, that $\epsilon' \geq 1/\mathsf{poly}(n)$. We then construct a malicious $\mathsf{WI\text{-}P}^*$ as follows: $\mathsf{WI\text{-}P}^*$ engages with $\mathsf{P}^*$ in a protocol execution where it impersonates the verifier and computes all of the messages honestly. Let $(\alpha, \beta, \gamma)$ be the variables determined by the transcript of the execution. Then $\mathsf{WI\text{-}P}^*$ checks that $\mathsf{SPS\text{-}V}(\tau, \alpha, \beta, \gamma) = 1$ and extracts the witness $u$ from $(\alpha, \beta, \gamma)$ in time $O(2^{n_{\mathsf{SPSZK}}})$ (recall the choice of parameters from Section 4.2) if this is the case. If the extraction fails or the transcript does not verify, then $\mathsf{WI\text{-}P}^*$ aborts. Finally, $\mathsf{WI\text{-}P}^*$ outputs $(x_{\mathsf{NIWI}}, \tau, \pi, u)$.

It is easy to see that $\mathsf{WI\text{-}P}^*$ perfectly simulates the verifier's preprocessing as well as the execution phase for $\mathsf{P}^*$. $\mathsf{WI\text{-}P}^*$ successfully cheats, if $(\tau, \pi)$ verifies, extraction is successful, and $x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}}$.

Note that $1 \leftarrow \left\langle \mathsf{P}^*(x^*, s_1, p_2), \mathsf{V}^1(x^*, s_2, p_1) \right\rangle$ implies that both $(\alpha, \beta, \gamma)$ as well as $(\tau, \pi)$ verify correctly. Assume for the moment that the extraction from $(\alpha, \beta, \gamma)$ is successful with probability $1 - \mathsf{negl}(n)$. Then it holds that

$$\begin{aligned}
&\Pr\left[(x_{\mathsf{NIWI}}, \tau, \pi, u) \leftarrow \mathsf{WI\text{-}P}^*(1^n) : \begin{matrix} x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}} \wedge \mathsf{WI\text{-}P}_1(u) = \tau \\ \wedge\ \mathsf{WI\text{-}V}(x_{\mathsf{NIWI}}, \tau, \pi) = 1 \end{matrix}\right] \\
\geq &\Pr\left[\mathsf{cheat} | x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}}\right] \cdot \Pr\left[x_{\mathsf{NIWI}} \notin \mathcal{L}_{\mathsf{NIWI}}\right] \cdot (1 - \mathsf{negl}(n)) \\
= &\epsilon' - \mathsf{negl}(n) = 1/\mathsf{poly}(n) - \mathsf{negl}(n).
\end{aligned}$$

Since $\mathsf{WI\text{-}P}^*$ runs in time $O(2^{n_{\mathsf{SPSZK}}}) + \mathsf{poly}(n)$ this would contradict the soundness of the HPP-NIWI. What is left to be shown is that the probability that the extraction from $(\alpha, \beta, \gamma)$ is not successful is bounded by a negligible function. If this was not the case, then $\alpha$ and the randomness used to compute it would uniquely determine $\beta$ (recall the properties of SPS-ZK from Section 2). Therefore we could find the randomness in time $O(2^{n_{\mathsf{SPSZK}}}) + \mathsf{poly}(n)$ and use it together with $\alpha$, to break the hiding property of $c = \mathsf{Com}(\beta)$. It follows that the extraction must succeed with all but negligible probability. We can conclude that $\epsilon' \leq \mathsf{negl}(n)$.

**Bounding $\epsilon''$.** Assume towards contradiction that $\epsilon'' \geq 1/\mathsf{poly}(n)$. Since $x^* \notin \mathcal{L}$, the definition of $\mathcal{L}_{\mathsf{NIWI}}$ implies that for an $x_{\mathsf{NIWI}} \in \mathcal{L}_{\mathsf{NIWI}}$ there exists an $(r, \tilde{r})$ such that $\mathsf{Com}(1; r) = c_0$ and $\mathsf{Com}(\tilde{w}; \tilde{r}) = \tilde{c}$. However, we can show that this would allow us to decide $\tilde{\mathcal{L}}$ in the average case as follows.

Given a random instance $\tilde{x}$, compute a verifier preprocessing honestly using $\tilde{x}$ as the random instance of the average-case hard language. The prover $\mathsf{P}^*$ returns its pre-processing and the commitment $\tilde{c}$. Then extract the content of $\tilde{c}$ in time $O(2^{n_{\mathsf{COM}}})$. If it contains a valid witness for $\tilde{x}$ return 1, else return a random bit. Note that if $\tilde{x} \notin \tilde{\mathcal{L}}$ then $\tilde{w}$ does not exist and therefore the algorithm described above will always output a random bit. On the other hand, if $\tilde{x} \in \tilde{\mathcal{L}}$ then we can lower bound the probability of the algorithm outputting 1 by $1/2 + \epsilon'' = 1/2 + 1/\mathsf{poly}(n)$. Since the described algorithm runs in time $O(2^{n_{\mathsf{COM}}}) + \mathsf{poly}(n)$ this clearly contradicts the average case hardness of $\tilde{\mathcal{L}}$ as specified in Section 4.2. We have thus established that $\epsilon = \epsilon' + \epsilon'' \leq \mathsf{negl}(n)$ and SF-ZK is therefore computationally sound.

**From selective to adaptive** . For the second step of the proof, we rely on complexity leveraging. Let $l_x$ be the domain size of the statement $l_x = |x|$. Let $\mathcal{B}$ against the adaptive security. We set $l_x$ to be

$$2^{l_x} \ll 2^{n_{\mathsf{SPSZK}}} \ll 2^{n_{\mathsf{OWF}}} \ll 2^{n_{\mathsf{COM}}} \ll 2^{n_{\mathsf{NIWI}}} = 2^{n_{\mathsf{L}}}.$$

We construct a reduction which behaves identically as the non-adaptive case, except that it guesses a statement $x$ and aborts if $x \neq x^*$. The analysis is identical to what described above, except that the advantage drops by a factor at most $1/2^{l_x}$.

**Theorem 3 (Observer Soundness).** *If* $(\mathsf{WI\text{-}P}, \mathsf{WI\text{-}V})$ *is an HPP-NIWI with unique proofs,* $\tilde{\mathcal{L}}$ *is an average-case hard language with unique witnesses,* $\mathsf{Com}$ *is a perfectly binding commitment scheme with unique openings, and* $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ *is an SPS-ZK argument, then the argument system SF-ZK in Figure 1 is observer sound.*

*Proof.* We describe the observer algorithm in Figure 2. Recall that the observer soundness definition considers two cases. In one case the prover acts honestly during the pre-processing phase ($\tilde{\mathsf{P}} = \mathsf{P}^1$), in the other case the verifier does ($\tilde{\mathsf{V}} = \mathsf{V}^1$). We analyze the two cases separately.

**Honest $\mathsf{P}^1$.** Assume towards contradiction, that there exists an $x^* \notin \mathcal{L}$, a malicious prover $\mathsf{P}^*$, and a malicious verifier $\mathsf{V}^*$ such that

$$\frac{1}{\mathsf{poly}(n)} \leq \Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \mathsf{P}^1(1^n), (s_2, p_2) \leftarrow \mathsf{V}^*(x^*), \\ \mathcal{T} := \langle \mathsf{P}^*(x^*, s_1, p_2), \mathsf{V}^*(x^*, s_2, p_1) \rangle\end{matrix} : \Theta(p_1, p_2, \mathcal{T}, x) = 1\right].$$

$\Theta(p_1, p_2, \mathcal{T}, x)$

1 : **if** $\mathsf{Com}(\beta; t) \neq c$ **or** $(\tilde{x}, \tilde{w}) \notin \tilde{\mathcal{R}}$
2 : **return** 0
3 : **elseif** $\mathsf{SPS\text{-}V}(\tau, \alpha, \beta, \gamma) = 0$ **or** $\mathsf{WI\text{-}V}((x, y, \tilde{w}, c_0, \bar{c}, \tilde{c}), \tau, \pi) = 0$
4 : **return** 0
5 : **else return** 1

**Fig. 2.** The observer algorithm $\Theta$

From this it follows that

$$\frac{1}{\mathsf{poly}(n)} \leq \Pr\left[(r, \tau) \leftarrow \mathsf{P}^1(1^n), \pi \in \mathcal{T} : \mathsf{WI\text{-}V}((x, y, \tilde{w}, c_0, \bar{c}, \tilde{c}), \tau, \pi) = 1\right]. \tag{1}$$

Where Equation 1 stems from the fact that the prover's pre-processing is honest and the observer always verifies the proof $\pi$. Recall that the statement $(x, y, \tilde{w}, c_0, \bar{c}, \tilde{c}) \in \mathcal{L}_{\mathsf{NIWI}}$ if and only if

$$\begin{aligned} &\exists(s, \tilde{r}) : (\underline{x^* \in \mathcal{L}} \wedge \mathsf{Com}(\tilde{r}; s) = \bar{c} \wedge \mathsf{Com}(0^n, \tilde{r}) = \tilde{c}) \\ \vee\ &\exists(r, \tilde{r}) : (\underline{\mathsf{Com}(1; r) = c_0} \wedge \mathsf{Com}(\tilde{w}, \tilde{r}) = \tilde{c}) \\ \vee\ &\exists(r, z) : (x^* \in \mathcal{L} \wedge \underline{\mathsf{Com}(1; r) = c_0} \wedge f(z) = y) \end{aligned} \tag{2}$$

By assumption, $x^* \notin \mathcal{L}$ and $\mathsf{Com}(0; r) = c_0$, since the prover's pre-processing is generated honestly and the commitment scheme is perfectly binding. Therefore each of the parts underlined in Equation 2 is false. By extensions, this makes the conjunction in each of the three branches false. It follows then that $\pi$ is a proof for a false statement given an honestly generated $\tau$, which contradicts the soundness of the HPP-NIWI.

**Honest $\mathsf{V}^1$.** For this case we can bootstrap the verifier's honest preprocessing into a fully honest verifier execution and then simply reduce observer soundness to regular soundness.

Assume towards contradiction that there exists an $x^* \notin \mathcal{L}$, a malicious prover $\mathsf{P}^*$, and a malicious verifier $\mathsf{V}^*$ such that

$$\frac{1}{\mathsf{poly}(n)} \leq \Pr\left[(s_1, p_1) \leftarrow \mathsf{P}^*(x^*), (s_2, p_2) \leftarrow \mathsf{V}^1(1^n), \mathcal{T} := \langle \mathsf{P}^*(x^*, s_1, p_2), \mathsf{V}^*(x^*, s_2, p_1)\rangle : \Theta(p_1, p_2, \mathcal{T}, x) = 1\right].$$

From this it follows that

$$\begin{aligned} \frac{1}{\mathsf{poly}(n)} &\leq \Pr\left[\begin{array}{l}(s_1, p_1) \leftarrow \mathsf{P}^*(x^*), \\ (s_2, p_2) \leftarrow \mathsf{V}^1(1^n), \\ \mathcal{T} := \langle \mathsf{P}^*(x^*, s_1, p_2), \mathsf{V}^2(x^*, s_2, p_1)\rangle\end{array} : \Theta(p_1, p_2, \mathcal{T}, x) = 1\right] && (3) \\ &= \Pr\left[\begin{array}{l}(s_1, p_1) \leftarrow \mathsf{P}^*(x^*), \\ (s_2, p_2) \leftarrow \mathsf{V}^1(1^n)\end{array} : 1 \leftarrow \langle \mathsf{P}^*(x^*, s_1, p_2), \mathsf{V}^2(x^*, s_2, p_1)\rangle\right] && (4) \end{aligned}$$

To see why Equation 3 holds, first note that the commitment scheme is perfectly binding and the language $\tilde{\mathcal{L}}$ has unique witnesses. Since $\Theta$ verifies in line 1 that $(\beta, t)$ is a valid decommitment of $c$ and that $\tilde{w}$ is indeed a witness of $\tilde{x}$, it follows that given the verifier's honest pre-processing there exists only a unique verifier message that does *not* cause the observer to output 0. For every possible transcript of the interaction between $\mathsf{P}^*$ and $\mathsf{V}^*$ consider the following two possibilities. Either the message sent by $\mathsf{V}^*$ is exactly that unique message or it sends any other message. In the first case, the malicious verifier behaves identically to the honest verifier and replacing $V^*$ by $V^2$ does not change the resulting transcript or the output of $\Theta$ at all. In the latter case, $\Theta$ already outputs 0 for this transcript anyway and the only change could be that $\Theta$ now outputs 1. Thus we can conclude that the probability of $\Theta$ outputting 1 can only increase. Thus Equation 3 must hold.

To see that Equation 4 must hold we simply need to consider the checks performed by $\Theta$ in line 3. It's easy to see that $\Theta(p_1, p_2, \mathcal{T}, x) = 1$ implies that $\mathsf{SPS\text{-}V}(\tau, \alpha, \beta, \gamma) = 1$ and $\mathsf{WI\text{-}V}((x, y, \tilde{w}, c_0, \bar{c}, \tilde{c}), \tau, \pi) = 1$, since the protocols are public-coin (and therefore publicly verifiable). However, these coincide with all

checks performed by the honest verifier. Therefore, in an execution between the malicious prover and the honest verifier, the honest verifier accepts if and only if the transcript is accepted by the observer. Equation 4 therefore holds. We've thus shown that

$$\frac{1}{\mathsf{poly}(n)} \leq \Pr\left[\begin{matrix}(s_1,p_1)\leftarrow \mathsf{P}^*(x^*),\\ (s_2,p_2)\leftarrow \mathsf{V}^1(1^n)\end{matrix} : 1 \leftarrow \left\langle \mathsf{P}^*(x^*,s_1,p_2), \mathsf{V}^2(x^*,s_2,p_1)\right\rangle\right]$$

which would contradict the soundness of SF-ZK. Therefore, an $x^*$ and $\mathsf{P}^*$ as assumed above cannot exist and SF-ZK must also be (selective) observer sound. The proof for the adaptive observer sound is the same as above.

**Theorem 4 (Zero Knowledge).** *If* $(\mathsf{WI\text{-}P}, \mathsf{WI\text{-}V})$ *is an HPP-NIWIs with unique proofs,* $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ *is an SPS-ZK argument with unique last message,* $f$ *is a one-way function with efficiently checkable range, and* $\mathsf{Com}$ *is a perfectly binding and computationally hiding commitment scheme, then the argument system SF-ZK in Figure 1 is computationally zero knowledge.*

*Proof.* We specify the zero-knowledge simulator $\mathsf{Sim}$ in the following. The simulator keeps a record of its running time and aborts if the number of steps exceeds $2^n$.

1. During the preprocessing phase the simulator acts exactly like the honest prover, except that it commits to 1 in $c_0 \leftarrow \mathsf{Com}(1, r)$.
2. In the execution phase, it initializes a counter $i = 0$ and runs the following lookahead thread.
   (a) Commit to $0^n$ in $\tilde{c}$ using fresh randomness and send $\tilde{c}$.
   (b) As a response $\mathsf{V}^*$ either aborts or sends a response $(\tilde{w}, \beta, t)$.
   (c) If $i = 0$ check whether the verifier aborts or $(\tilde{x}, \tilde{w}) \notin \tilde{\mathcal{R}}$ and abort the whole simulation if any of these conditions are met, outputting whatever $\mathsf{V}^*$ outputs. Otherwise set $i := 1$ and return to step 2a.
   (d) If $i \neq 0$ check whether the verifier aborts or $(\tilde{x}, \tilde{w}) \notin \tilde{\mathcal{R}}$ and return to step 2a if this is the case. Otherwise set $i := i + 1$; if $i = 12n$ exit the loop, otherwise return to step 2a.
3. Let $T$ be the number of iterations of the previous loop. Let $\tilde{p} := 12n/T$. Then the simulator enters in the following loop up to $(n^2/\tilde{p})$-many times.
   (a) Use the alternative witness $\tilde{w}$ to compute $\tilde{c} := \mathsf{Com}(\tilde{w}; r^*)$, using fresh random coins $r^*$, and send $\tilde{c}$ to the verifier.
   (b) As a response $\mathsf{V}^*$ either aborts or sends a second message $(\tilde{w}, \beta, t)$.
   (c) If the verifier aborts or the second message is invalid, return to step 3a, else exit the loop.
4. If $n^2/\tilde{p}$ iterations were reached without a valid $\tilde{w}$ being output by the verifier, output $\mathsf{fail}$. Else use the alternative witness $(r, r^*)$ to compute $\pi$ using the second branch (2) of the HPP-NIWI proof and compute $\gamma$ honestly. Send $(\gamma, \pi)$ to the verifier.
5. The simulator outputs whatever $\mathsf{V}^*$ outputs.

We first bound the running time of the simulator and the probability of the simulator outputting $\mathsf{fail}$.

**Lemma 1.** $\mathsf{Sim}$ *runs in expected polynomial time in* $n$*.*

*Proof.* Let $p(n)$ be the probability that $\mathsf{V}^*$ outputs a well-formed response given $\tilde{c}$ computed as in step 2a. Observe that the work of the simulator is strictly polynomial time except for the number of rewindings, therefore it is sufficient to bound the number of iterations. Note that from [41] the expected number of iterations of the first loop is exactly $\frac{12n}{p(n)}$. With this observation in mind, we distinguish between two cases

1. $\frac{p(n)}{\tilde{p}} \neq O(1)$. In this case, we use the trivial bound $2^n$. However, this case can be shown to happen with negligible probability by the Chernoff bound.
2. $\frac{p(n)}{\tilde{p}} = O(1)$. In this case we can bound the running time by

$$\mathsf{poly}(n) \cdot p(n) \cdot \left(\frac{12n}{p(n)} + \frac{n^2}{\tilde{p}}\right) = \mathsf{poly}(n) \cdot \frac{p(n)}{\tilde{p}} = \mathsf{poly}(n)$$

which concludes our analysis.

Next we bound the probability that the simulator outputs $\mathsf{fail}$.

*Claim.* The probability that $\mathsf{Sim}$ outputs $\mathsf{fail}$ is negligible in $n$.

Let $q(n)$ be the probability that $\mathsf{V}^*$ outputs a well-formed response given $\tilde{c}$ (computed as in step 3). We state and prove the following helping lemma.

**Lemma 2.** *There exists a negligible function such that* $q(n) \geq p(n) - \mathsf{negl}(n)$.

*Proof.* If $p(n)$ is negligible than it is trivial. Else it can be easily shown via a two-step argument. Let us define $\mathfrak{q}(n)$ as $q(n)$ except that in the simulation the commitment $\bar{c}$ is computed as the commitment to a random string. Note that in the real protocol the corresponding opening $s$ is used only after the last message of $\mathsf{V}^*$ and therefore $q(n) = \mathfrak{q}(n) - \mathsf{negl}(n)$ by the hiding of the commitment scheme.

Recall that $p(n)$ is defined as the probability of $\mathsf{V}^*$ to abort given $\tilde{c} = \mathsf{Com}(0)$ using fresh randomness and $\mathfrak{q}(n)$ is defined as the probability of $\mathsf{V}^*$ to abort given $\tilde{c} = \mathsf{Com}(\tilde{w})$ using fresh randomness. Thus we can use $\mathsf{V}^*$ as a distinguisher for the commitment scheme and it will succeed with probability $p(n) - \mathfrak{q}(n)$. Since this value can be bound by a negligible function by the computational hiding of $\mathsf{Com}$, we have that

$$p(n) - (q(n) + \mathsf{negl}(n)) = p(n) - \mathfrak{q}(n) \leq \mathsf{negl}(n)$$

which implies that $q(n) \geq p(n) - \mathsf{negl}(n)$ and concludes our proof.

We are now in the position of proving our claim.

*Proof.* Recall that the simulator outputs $\mathsf{fail}$ if all $\frac{n^2}{\tilde{p}}$ iterations in step 3 are not successful. We consider two cases.

1. $p(n) \leq 2 \cdot \mathsf{negl}(n)$. In this case the simulator reaches step 3 with negligible probability and therefore $\mathsf{fail}$ happens with the same probability.
2. $p(n) > 2 \cdot \mathsf{negl}(n)$. For conceptual simplicity we split the loop in step 3 to $n$ independent rewinds, each upper-bounded by $\frac{n}{\tilde{p}}$ steps. Then $\mathsf{fail}$ happens if all of the rewinds as not successful. By a routine calculation we obtain that the expected number of iterations of each rewinding until a successful instance is found is
$$\frac{1}{q(n)} \leq \frac{1}{p(n) - \mathsf{negl}(n)} < \frac{2}{p(n)} = O\left(\frac{1}{\tilde{p}}\right),$$
where the first inequality is by Lemma 2 and last equality is discussed above. By Markov's inequality the probability that the simulator tries more than $\frac{n}{\tilde{p}}$ iterations is at most $O(1/n)$. Since we consider $n$ independent instances, the total probability is bounded by $O(1/n)^n$.

Finally, we show that the distribution induced by the output of the simulator is computationally indistinguishable from the honest one. Consider the following sequence of hybrids.

**Hybrid** $\mathsf{H}_1$: The first hybrid is the interaction between the simulator $\mathsf{Sim}$ and the malicious verifier $\mathsf{V}^*$.
**Hybrid** $\mathsf{H}_2$: The last message $\gamma$ of the SPS-ZK is simulated in time $O(2^{n_{\mathsf{SPSZK}}})$.
**Hybrid** $\mathsf{H}_3$: The simulator inverts the one-way function to obtain $\tilde{z}$ such that $f(\tilde{z}) = y$ and uses it, together with the original witness $w$ and the randomness $r$, to compute $\pi$ by satisfying the third branch (3).
**Hybrid** $\mathsf{H}_4$: The simulator computes $\tilde{c}$ as $\mathsf{Com}(0^n)$ using fresh random coins.
**Hybrid** $\mathsf{H}_5$: The simulator no longer rewinds the verifier and simply executes the protocol in a single thread.
**Hybrid** $\mathsf{H}_6$: The commitment $\tilde{c}$ is computed using the committed randomness $\tilde{r}$, instead of a fresh $r^*$.
**Hybrid** $\mathsf{H}_7$: The simulator computes $\pi$ using the original witness $(w, s, \tilde{r})$, without inverting $f$.
**Hybrid** $\mathsf{H}_8$: The SPS-ZK is no longer simulated and instead computed honestly.
**Hybrid** $\mathsf{H}_9$: The simulator now commits to 0 in $c_0 = \mathsf{Com}(0; r)$.

It is easy to see that the last hybrid exactly matches the honest execution. We will show that each $\delta_i$ defined as

$$\delta_i := |\Pr[\mathcal{D}(\langle \mathsf{H}_i(x, w), \mathsf{V}^*(x)\rangle) = 1] - \Pr[\mathcal{D}(\langle \mathsf{H}_{i+1}(x, w), \mathsf{V}^*(x)\rangle) = 1]|$$

is negligible in $n$. Note that the simulator $\mathsf{Sim}$ runs in expected (super) polynomial-time, whereas all of the following reductions must terminate in strict (super) polynomial-time. This issue can be dealt with

by truncating $\mathsf{Sim}$ to twice its expected running time. By Markovs inequality, this reduces its success probability by at most $1/2$.

Observe that the difference in the first hybrid is that the SPS-ZK protocol is simulated in super-polynomial time. The simulator simply guesses the challenge of the verifier ahead of time and restarts the whole execution if the guess was not correct. The expected number of attempts is in the order of $O(2^{n_{\mathsf{SPSZK}}})$, however, when the simulator is successful, the transcript of the execution is statistically close to the transcript of an honest run. This bounds the value of $\delta_1$ (and analogously of $\delta_7$) to a negligible function.

The differences $\delta_2$ and $\delta_6$ can be shown to be negligible with a reduction to the witness indistinguishability of the HPP-NIWI arguments: The reduction simply sets $\pi$ to be the challenge proof and returns the output of the distinguisher. Note that the random coins of the setup are not required for the simulation. The reduction runs in time $O(2^{n_{\mathsf{OWF}}}) + O(2^{n_{\mathsf{SPSZK}}}) + \mathsf{poly}(n)$ and therefore the differences among these hybrids can be bound by a negligible function (recall the parameter setup from Section 4.2).

Note that the fifth hybrid differs from the fourth only in case $\mathsf{fail}$ happens, however by Lemma 4.2 this happens with negligible probability and the bound on $\delta_4$ follows. $\delta_3$ and $\delta_5$ can be shown to be negligible with a trivial reduction to the hiding property of the commitment scheme. Note that the reduction runs in time $O(2^{n_{\mathsf{OWF}}}) + O(2^{n_{\mathsf{SPSZK}}}) + \mathsf{poly}(n)$, however the commitment scheme is assumed to be hiding for machines bounded by such a runtime. The bound on $\delta_8$ uses an identical argument except that now the reduction runs in (strict) polynomial time. We can conclude that

$$|\Pr[\mathcal{D}(\langle \mathsf{P}(x,w), \mathsf{V}^*(x)\rangle) = 1] - \Pr[\mathcal{D}(\mathsf{Sim}(x)) = 1]| \leq \sum_{i=1}^{9} \delta_i \leq \mathsf{negl}(n)\,.$$

**Theorem 5 (Computationally Unique Transcripts).** *If* $(\mathsf{WI\text{-}P}, \mathsf{WI\text{-}V})$ *is an HPP-NIWI with unique proofs,* $\tilde{\mathcal{L}}$ *is an average-case hard language with unique witnesses,* $f$ *is a one-way function,* $(\mathsf{SPS\text{-}P}, \mathsf{SPS\text{-}V})$ *is an SPS-ZK argument with unique last messages, and the commitment scheme* $\mathsf{Com}$ *is perfectly binding and has unique openings, then the argument system SF-ZK in Figure 1 has computationally unique transcripts.*

*Proof.* Recall that a pair of machines $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$ is admissible if at least one of the two is identical to an honest generation algorithm. We treat the two cases separately.

**Honest $\mathsf{P}^1$.** First observe that the verifier only sends the decommitment $(\beta, t)$ and the witness $\tilde{w}$. Since the commitment scheme is perfectly binding and has unique decommitments and $\Theta$ verifies that the decommitment is correct, then $(\beta, t)$ is uniquely determined by the preprocessing. Further, $\tilde{\mathcal{L}}$ has unique witnesses, therefore $\tilde{w}$ is also fixed by the preprocessing, for any choice of $\tilde{x}$.

On the prover's side the tuple $(\tilde{c}, \gamma, \pi)$ collects all messages sent in the execution. Since the prover's preprocessing phase is honest, $c_0$ is a commitment to 0. Since the commitment scheme is perfectly binding and has unique decommitments, then $\bar{c}$ from the pre-processing fixes both $(s, \tilde{r})$. If we assume towards contradiction that there exists two different accepting $\tilde{c}$ and $\hat{c}$, by the soundness of $\pi$ we have that $\tilde{c} = \mathsf{Com}(0^n, \tilde{r})$ and $\hat{c} = \mathsf{Com}(0^n, \hat{r})$, where $\bar{c} = \mathsf{Com}(\tilde{r}, s)$ and $\bar{c} = \mathsf{Com}(\hat{r}, s)$. However this is a contradiction since the commitment has unique openings. It follows that $\tilde{r} = \hat{r}$ and therefore $\tilde{c}$ is unique. Recall that both the HPP-NIWI and the SPS-ZK have unique last messages, and therefore $(\gamma, \pi)$ are uniquely determined by the pre-processing.

**Honest $\mathsf{V}^1$.** Given an honest verifier pre-processing $p_2$ and a (possibly malicious) prover pre-processing $p_1$ for a certain statement $x$ with unique witnesses, let $\mathcal{T}_1 := (\tilde{c}, \beta, t, \tilde{w}, \pi, \gamma)$ and $\mathcal{T}_2 := (\hat{c}, \hat{\beta}, \hat{t}, \hat{w}, \hat{\pi}, \hat{\gamma})$ be the two transcripts such that $\Theta(p_1, p_2, \mathcal{T}_1, x) = \Theta(p_1, p_2, \mathcal{T}_2, x) = 1$. We shall prove that $\mathcal{T}_1 = \mathcal{T}_2$ with all but negligible probability.

$\underline{(\beta, t) = (\hat{\beta}, \hat{t})}$ : Since the commitment scheme is perfectly binding and has unique openings and $c = \mathsf{Com}(\beta; t)$ is fixed in the pre-processing, this equality must hold.

$\underline{\tilde{w} = \hat{w}}$ : The witness is uniquely determined by the statement $\tilde{x}$, since $\tilde{\mathcal{L}}$ has unique witnesses.

$\underline{\gamma = \hat{\gamma}}$ : Fix $\tau$ and $(\alpha, \beta)$, which are all part of the pre-processing, then $\gamma$ is unique since the SPS-ZK has unique last messages.

$\underline{\pi = \hat{\pi}}$ : First note that there must exist some $u$ such that $\mathsf{WI\text{-}P}_1(u) = \tau$. To see why this is the case, recall either the transcript of the SPS-ZK proof uniquely determines the witness or $\alpha$ (together with the randomness used to compute it) uniquely determines the challenge $\beta$. If a valid $u$ does not exist, then we are left with the latter case, which implies that we can guess the content of $c$ running in time $O(2^{n_{\mathsf{SPSZK}}})$.

This contradicts the hiding property of $\mathsf{Com}$ (refer to Section 2 for further discussion). It follows that $\tau$ is well-formed except with negligible probability.

Therefore both $\pi$ and $\hat{\pi}$ are generated using the witness for one of the following branches:

$$\begin{aligned} & \exists(s,\tilde{r}) : (x^* \in \mathcal{L} \wedge \mathsf{Com}(\tilde{r};s) = \bar{c} \wedge \mathsf{Com}(0^n,\tilde{r}) = \tilde{c}) \\ \vee\ & \exists(r,\tilde{r}) : (\mathsf{Com}(1;r) = c_0 \wedge \mathsf{Com}(\tilde{w},\tilde{r}) = \tilde{c}) \\ \vee\ & \exists(r,z) : (x^* \in \mathcal{L} \wedge \mathsf{Com}(1;r) = c_0 \wedge f(z) = y) \end{aligned}$$

We bound the probability that $\pi$ or $\hat{\pi}$ is a valid proof for the second branch (2) in the following. Assume without loss of generality that such proof is $\pi$. Since the commitment scheme is perfectly binding we can extract $\tilde{w}$ from $\tilde{c}$ (by exhaustive search) in time $O(2^{n_{\mathsf{COM}}})$. Note that the extraction is successful with probability 1 since $\tilde{c}$ is perfectly binding. Recall that $\tilde{\mathsf{V}}^1$ is honest by assumption and therefore we can plug in a hard instance $\tilde{x}$ and break the average-case hardness of $\tilde{\mathcal{L}}$ from the first message of the prover.

On the other hand, if the third branch (3) is proven with non-negligible probability then we can invert $y$ in time $O(2^{n_{\mathsf{SPSZK}}}) + \mathsf{poly}(n)$ by extracting $u$ from $(\alpha,\beta,\gamma)$ and running the polynomial-time extractor of the HPP-NIWI proof. It follows that both proofs are for the first branch (1), which implies that they are identical.

$\underline{\tilde{c} = \hat{c}:}$ As we argued above, $\pi$ must be a proof for the first branch. Since $(s,\tilde{r})$ are fixed in the pre-processing by $\bar{c}$, then $\tilde{c}$ is also uniquely determined, unless $\pi$ is a proof for a false statement. This happens only with negligible probability.

## 5 Multi-Execution Steganography-Free ZK

In the following, we extend our definition of steganography-free zero-knowledge to the multi-execution setting. In this setting, the prover and the verifier sample and output a fresh pre-processing (composed of a secret and a public part) during every execution phase, thereby "refreshing" the pre-processing. Since both the prover and the verifier may be malicious during the execution phase, the main challenge is to ensure that the newly sampled pre-processing is not fully malicious.

We begin by defining recursively the notion of admissible pre-processing.

**Definition 9 (Admissible Pre-Processing).** *A tuple $(s_1,p_1,s_2,p_2)$ is an* admissible pre-processing *if it holds that*

- $(s_1,p_1) \leftarrow \mathsf{P}^1(1^n)$, *or*
- $(s_2,p_2) \leftarrow \mathsf{V}^1(1^n)$, *or*
- *there exists some $(x,w)$ such that*

$$(1,(s_1,p_1),(s_2,p_2)) \leftarrow \langle \mathsf{P}^*(\tilde{s}_1,\tilde{p}_2,x,w), \mathsf{V}^*(\tilde{s}_2,\tilde{p}_1,x)\rangle \wedge \Theta(\tilde{p}_1,\tilde{p}_2,\mathcal{T},x) = 1$$

*where $(\tilde{s}_1,\tilde{p}_1,\tilde{s}_2,\tilde{p}_2)$ is an admissible pre-processing, $\mathcal{T}$ is the transcript of the latter interaction. The number of recursion is polynomially bounded.*

Observe that the definition of admissible pre-processing does not constrain the behavior of the parties except that we require that either of the very first pre-processing is honestly executed. As a consequence, a pre-processing that is the output of a protocol execution is admissible as long as the previous one was. This allows us to condition all of the requirements to the fact that the pre-processings are admissible, which lifts the definitions to the multi-execution settings.

**Definition 10 (Multi-Execution Steganography-Free Arguments).** *Let $\mathcal{L}$ be a language in NP with corresponding relation $\mathcal{R}$. An interactive argument with pre-processing $\Pi = (\mathsf{P}^1,\mathsf{P}^2,\mathsf{V}^1,\mathsf{V}^2)$ for language $\mathcal{L}$ is a multi-execution steganography-free protocol with pre-processing if the following conditions are satisfied.*

$\underline{\text{Completeness.}}$ *For all $(x,w) \in \mathcal{R}$ and all admissible $(s_1,p_1,s_2,p_2)$ it holds that*

$$\Pr\left[(1,\cdot,\cdot) \leftarrow \left\langle \mathsf{P}^2(x,w,s_1,p_2), \mathsf{V}^2(x,s_2,p_1)\right\rangle\right] \geq 1 - \mathsf{negl}(n)\,.$$

$\underline{\text{Non-Adaptive Computational Soundness.}}$ *For all $x \notin \mathcal{L}$, all admissible $(s_1,p_1,s_2,p_2)$, and all malicious* $\mathsf{PPT}$ *provers $\mathsf{P}^*$ it holds that*

$$\Pr\left[(1,\cdot,\cdot) \leftarrow \left\langle \mathsf{P}^*(x,s_1,p_2), \mathsf{V}^2(x,s_2,p_1)\right\rangle\right] \leq \mathsf{negl}(n)\,.$$

*Computational Soundness.* *For all admissible $(s_1, p_1, s_2, p_2)$, and all malicious* PPT *provers* $\mathsf{P}^*$ *it holds that*

$$\Pr\left[x \leftarrow \mathsf{P}^*(s_1, p_2); (1, \cdot, \cdot) \leftarrow \left\langle \mathsf{P}^*(x, s_1, p_2), \mathsf{V}^2(x, s_2, p_1)\right\rangle \wedge x \notin \mathcal{L}\right] \leq \mathsf{negl}(n).$$

*Notice that, we use the terms computational soundness and adaptive computational soundness interchangeably.*

*Zero-Knowledge.* *For all malicious* PPT *verifiers* $\mathsf{V}^*$ *there exists an expected polynomial time simulator* $\mathsf{Sim}$*, such that for all* PPT *distinguishers* $\mathcal{D}$*, it holds that for all polynomials* $m = m(n)$*, all tuples* $(w_1, x_1), \ldots, (w_m, x_m) \in \mathcal{R}$

$$\left|\Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \mathsf{P}^1(1^n),\\ (s_2, p_2) \leftarrow \mathsf{V}^*(x_1, \ldots, x_m)\end{matrix} : \mathcal{D}(\mathcal{T}_{\mathsf{Honest}}) = 1\right] - \Pr\left[\begin{matrix}(s_1, p_1) \leftarrow \mathsf{Sim}(1^n),\\ (s_2, p_2) \leftarrow \mathsf{V}^*(x_1, \ldots, x_m)\end{matrix} : \mathcal{D}(\mathcal{T}_{\mathsf{Sim}}) = 1\right]\right| \leq \mathsf{negl}(n),$$

*where* $\mathcal{T}_{\mathsf{Honest}}$ *denotes the transcript of the (sequential) interaction between the honest prover (on input* $(w_1, x_1), \ldots, (w_m, x_m)$*) and the (possibly corrupted) verifier, whereas* $\mathcal{T}_{\mathsf{Sim}}$ *denotes the transcript of the simulator (on input* $x_1, \ldots, x_m$*) with the (possibly corrupted) verifier.*

*Observer Completeness.* *For all* $(x, w) \in \mathcal{R}$ *and all admissible* $(s_1, p_1, s_2, p_2)$ *it holds that*

$$\Pr\left[\mathcal{T} := \left\langle \mathsf{P}^2(s_1, p_2, x, w), \mathsf{V}^2(s_2, p_1, x)\right\rangle : \Theta(p_1, p_2, \mathcal{T}, x) = 1\right] \geq 1 - \mathsf{negl}(n).$$

*Non-Adaptive Observer Soundness.* *For all* $x \notin \mathcal{L}$*, all admissible* $(s_1, p_1, s_2, p_2)$*, all* PPT *algorithms* $\mathsf{P}^*$ *and* $\mathsf{V}^*$ *it holds that*

$$\Pr\left[\mathcal{T} := \langle \mathsf{P}^*(s_1, p_2, x, w), \mathsf{V}^*(s_2, p_1, x)\rangle : \Theta(p_1, p_2, \mathcal{T}, x) = 1\right] \leq \mathsf{negl}(n).$$

*Observer Soundness.* *For all admissible* $(s_1, p_1, s_2, p_2)$*, all* PPT *algorithms* $\mathsf{P}^*$ *and* $\mathsf{V}^*$ *it holds that*

$$\Pr\left[x \leftarrow \mathsf{P}^*(s_1, p_2); \mathcal{T} := \langle \mathsf{P}^*(s_1, p_2, x, w), \mathsf{V}^*(s_2, p_1, x)\rangle : \begin{matrix}\Theta(p_1, p_2, \mathcal{T}, x) = 1\\ \wedge\, x \notin \mathcal{L}\end{matrix}\right] \leq \mathsf{negl}(n).$$

*Notice that, we use the terms observer soundness and adaptive observer soundness interchangeably.*

*Computationally Unique Transcripts.* *For all* $x \in \mathcal{L}$ *such that there exists a unique* $w$ *such that* $\mathcal{R}(x, w) = 1$*, all admissible* $(s_1, p_1, s_2, p_2)$*, and all* PPT *algorithms* $\left(\mathsf{P}^*, \mathsf{V}^*, \hat{\mathsf{P}}^*, \hat{\mathsf{V}}^*\right)$ *it holds that*

$$\Pr\left[\begin{matrix}\mathcal{T}_1 := \langle \mathsf{P}^*(s_1, p_2, x, w), \mathsf{V}^*(s_2, p_1, x)\rangle,\\ \mathcal{T}_2 := \left\langle \hat{\mathsf{P}}^*(s_1, p_2, x, w), \hat{\mathsf{V}}^*(s_2, p_1, x)\right\rangle\end{matrix} : \begin{matrix}\Theta(p_1, p_2, \mathcal{T}_1, x) = 1 \wedge\\ \Theta(p_1, p_2, \mathcal{T}_2, x) = 1 \wedge\\ \mathcal{T}_1 \neq \mathcal{T}_2\end{matrix}\right] \leq \mathsf{negl}(n).$$

### 5.1 Our Protocol

Let $\mathsf{Com}$ be a perfectly binding commitment scheme. Let $\mathsf{PRF}$ be a pseudorandom function. Let $(\mathsf{KGen}, \mathsf{Enc}, \mathsf{Eval}, \mathsf{Dec})$ be a levelled fully homomorphic encryption scheme. Let $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$ be an SF-ZK protocol for the following language

$$\mathcal{L}_{\mathsf{Full}} = \{(x, c, d) | \exists (w, \mathsf{sk}, \rho) : \mathcal{R}(x, w) = 1 \wedge \mathsf{Dec}(\mathsf{sk}, c) = 1 \wedge d = \mathsf{Com}(\mathsf{sk}; \rho)\}.$$

Let $(\mathsf{P}_{\mathsf{Dec}}, \mathsf{V}_{\mathsf{Dec}})$ be an SF-ZK protocol for the following language

$$\mathcal{L}_{\mathsf{Dec}} = \{(c, d) | \exists (\mathsf{sk}, \rho) : \mathsf{Dec}(\mathsf{sk}, c) = 1 \wedge d = \mathsf{Com}(\mathsf{sk}; \rho)\}.$$

For convenience we define the functions $\mathsf{PrePro}$ and $\mathsf{EqCheck}_{i,m}$ as

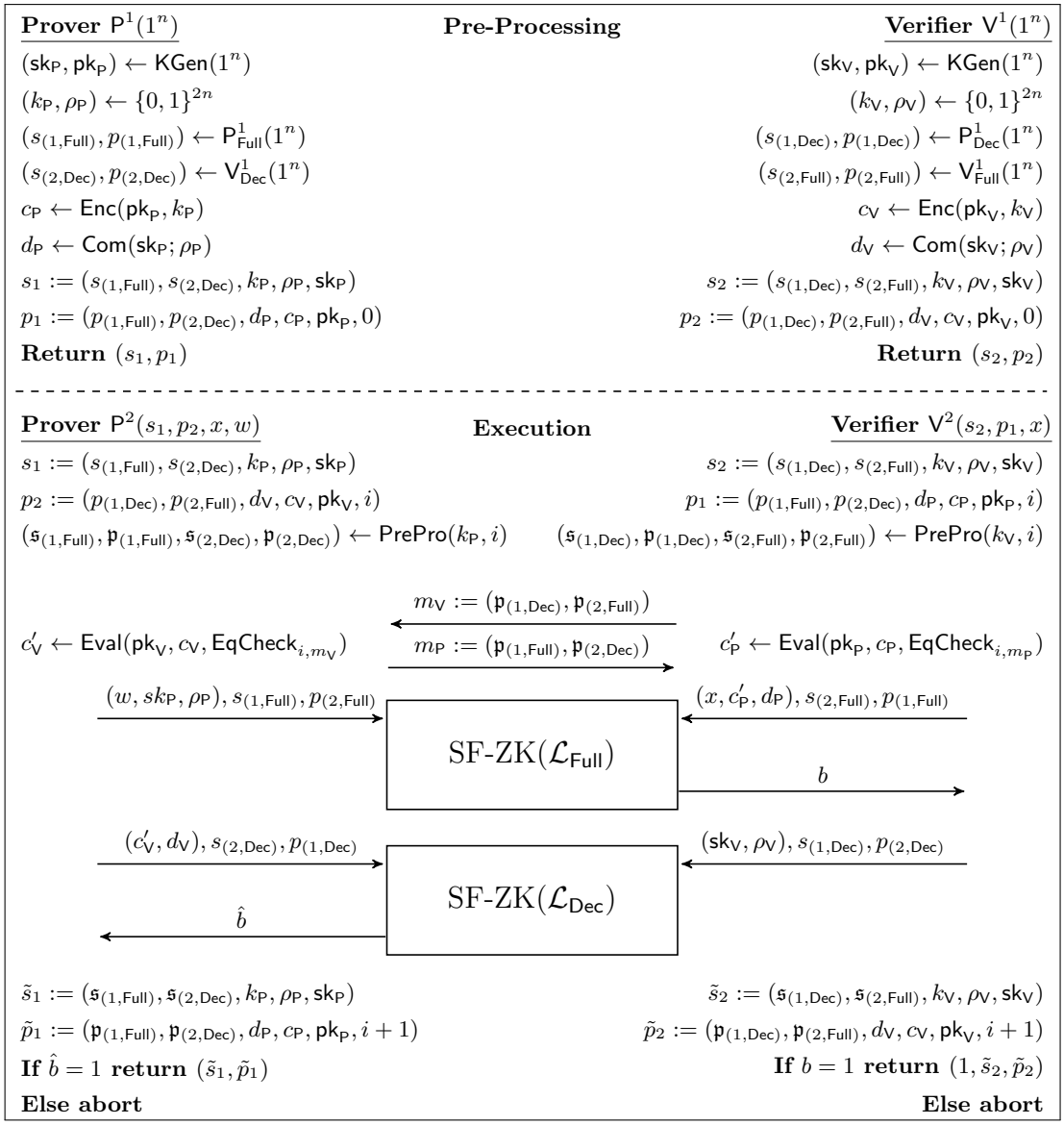


**Fig. 3.** Our multi-execution SF-ZK protocol.

$\underline{\mathsf{PrePro}(k, i):}$
$r_0 \leftarrow \mathsf{PRF}(k, i\|0)$
$r_1 \leftarrow \mathsf{PRF}(k, i\|1)$
**return** $(\mathsf{P}^1(1^n; r_0), \mathsf{V}^1(1^n; r_1))$

$\underline{\mathsf{EqCheck}_{i,m}(k):}$
$(s_\mathsf{P}, p_\mathsf{P}, s_\mathsf{V}, p_\mathsf{V}) \leftarrow \mathsf{PrePro}(k, i)$
**return** $(p_\mathsf{P}, p_\mathsf{V}) = m$.

Our construction is a generic compiler that turns any single-execution SF-ZK into a multi-execution SF-ZK'. The pre-processing procedure and the execution phase of SF-ZK' are shown in Figure 3.

**Pre-Processing.** In the one-time pre-processing phase, the prover $\mathsf{P}$ initializes a key pair $(\mathsf{sk}_\mathsf{P}, \mathsf{pk}_\mathsf{P})$ for an FHE scheme and a PRF key $k_\mathsf{P}$. Then it encrypts $k_\mathsf{P}$ under their public key $\mathsf{pk}_\mathsf{P}$ (producing the ciphertext $c_\mathsf{P}$) and computes a commitment $d_\mathsf{P}$ to $\mathsf{sk}_\mathsf{P}$. Finally, it computes the prover's pre-processing for the SF-ZK for the language $\mathcal{L}_{\mathsf{Full}}$ and the verifier's pre-processing for the SF-ZK for the language $\mathcal{L}_{\mathsf{Dec}}$. Its output consists of the pre-processing information of both SF-ZK instances, together with the ciphertext $c$, the commitment $d_\mathsf{P}$, the keys $(\mathsf{sk}_\mathsf{P}, \mathsf{pk}_\mathsf{P})$, and the random coins sampled in this phase. The pre-processing of the verifier $\mathsf{V}$ is identical, except that it plays the role of the verifier in the SPS-ZK

for $\mathcal{L}_{\mathsf{Full}}$ and the role of the prover in the SPS-ZK for $\mathcal{L}_{\mathsf{Dec}}$. Looking ahead, the execution of the SF-ZK protocols will ensure that the refreshed pre-processing is computed correctly by both parties.

**Execution.** In the execution phase, the prover $\mathsf{P}$ computes a fresh pre-processing for the SF-ZK arguments using pseudorandom coins (derived from the corresponding key $k_{\mathsf{P}}$). In addition to that, it also computes the pre-processing of the verifier homomorphically, using the corresponding ciphertext $c_{\mathsf{V}}$ (which contains $k_{\mathsf{V}}$). Then it executes the SF-ZKs to prove that its pre-processing is well-formed and to verify that the verifier's pre-processing is correctly generated. Note that the latter consists of a proof of correct decryption since the prover locally computes the equality check homomorphically. Here the usage of FHE is crucial to avoid the exponential blowup that would result from nesting the proofs of well-formedness of subsequent pre-processing. The verifier $\mathsf{V}$ also runs the same steps in parallel.

At this point, both parties are convinced that the refreshed pre-processing is computed correctly. To additionally prove that $x \in \mathcal{L}$ we include a conjunctive clause in the argument of the prover, then the verifier will automatically abort if this is not the case. Note that all operations, in addition to the SF-ZK executions, are deterministic and therefore the uniqueness of the transcript is preserved by our transformation.

### 5.2 Analysis

In the part, we show that the protocol satisfies all of the properties of a multi-execution SF-ZK.

**Theorem 6 (Multi-Execution Steganography-Free ZK).** *If* $\mathsf{Com}$ *is a perfectly binding commitment scheme,* $\mathsf{PRF}$ *is a pseudorandom function,* $(\mathsf{KGen}, \mathsf{Enc}, \mathsf{Eval}, \mathsf{Dec})$ *is a CPA-secure fully homomorphic encryption scheme,* $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$ *and* $(\mathsf{P}_{\mathsf{Dec}}, \mathsf{V}_{\mathsf{Dec}})$ *are SF-ZK protocols, then the argument system presented in Figure 3 is a multi-execution SF-ZK protocol.*

*Proof.* We argue about each property separately.

**Correctness.** The protocol is correct if the commitment, the PRF, the FHE and the SF-ZK protocols are correct.

**Observer Correctness.** The observer runs the observer algorithm for each SF-ZK protocol ($\Theta_{\mathsf{Full}}$ and $\Theta_{\mathsf{Dec}}$), in addition, it checks that the ciphertexts $(c'_{\mathsf{P}}, c'_{\mathsf{V}})$ are correctly generated, the correct public parameters are used in subsequent executions of the protocol, and the counter $i$ is increased by 1 after each iteration. If all of the above conditions are satisfied, the observer returns 1, else it returns 0. If both parties behave honestly, the observer returns 1 with probability 1.

**Soundness.** The soundness of the first iteration of the protocol is trivial. To argue about the $i$-th iteration we define the following sequence of hybrids.

**Hybrid** $\mathsf{H}_1$: Defined as the honest execution.
**Hybrid** $\mathsf{H}_2$: The verifier uses the simulator to execute the protocol $\mathsf{P}_{\mathsf{Dec}}$.
**Hybrid** $\mathsf{H}_3$: The commitment $d_{\mathsf{V}}$ is computed as commitment to 0.
**Hybrid** $\mathsf{H}_4$: The ciphertext $c_{\mathsf{V}}$ is computed as $\mathsf{Enc}(\mathsf{pk}_{\mathsf{V}}, 0)$.
**Hybrid** $\mathsf{H}_5$: The pair $(\mathfrak{s}_{(2,\mathsf{Full})}, \mathfrak{p}_{(2,\mathsf{Full})})$ is computed using a uniformly sampled string, instead of the output of the $\mathsf{PRF}$.

The distance between each pair of hybrids can be reduced to the zero-knowledge of the ZK protocol, the hiding of the commitment scheme, the CPA-security of the FHE, and the security of a PRF, respectively. Note that in hybrid 5 the secret string $\mathfrak{s}_{(2,\mathsf{Full})}$ of the $(i-1)$-th iteration is information-theoretically hidden from the eyes of the adversary.

We are now in the position of arguing about the soundness of the $i$-th run. Assume for the moment that the public parameters of the $i$-th instance of the prover for the ZK protocol $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$ are honestly generated. Then soundness of the $i$-th iteration follows from the soundness of $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$. If we assume the contrary, then this implies that the public parameters of the $i$-th session are not honestly generated, which in turn implies the existence of an adversary the breaks the soundness of the $(i-1)$-th instance of $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$. By induction, the adversary has to break the soundness of the first iteration of the protocol, where the parameters are honestly generated. This is a contradiction.

**Observer Soundness.** Without loss of generality we consider the case of a corrupted verifier pre-processing, the other case follows along the same lines. The observer soundness of the first iteration follows directly from the observer soundness of $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$: Since the public parameters $p_{1,\mathsf{Full}}$ are honestly generated and the statement of the SF-ZK protocol is fixed once $x$ is fixed, then it is computationally hard to find a transcript $\mathcal{T}$ that causes the observer to accept a false proof for $\mathcal{L}_{\mathsf{Full}}$. The same holds for the $i$-th iteration if the corresponding public parameters are honestly generated. We are going to show this with an induction on the number of iterations.

For the base case, let $\mathfrak{p}_{1,\mathsf{Full}}$ be the public parameters that are output by the first iteration. Assume (towards contradiction) that they are not honestly generated but the observer outputs 1. By the correctness of the FHE scheme and by the observer soundness of $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$, then it must be the case that $\Theta_{\mathsf{Full}}$ returns 0. Since the observer runs $\Theta_{\mathsf{Full}}$ as a subroutine, this is a contradiction.

By induction hypothesis, $\mathfrak{p}_{1,\mathsf{Full}}$ output by the $(i-1)$-th iteration are correctly generated. The same argument as above shows that the public parameters output by the $i$-th iteration are also correctly generated, or the observer returns 0.

**Zero Knowledge.** To argue about zero-knowledge we define a series of hybrid experiments.

**Hybrid** $\mathsf{H}_1$: Defined as the honest execution.
**Hybrid** $\mathsf{H}_2$: The verifier uses the simulator to execute the protocol $\mathsf{P}_{\mathsf{Full}}$.
**Hybrid** $\mathsf{H}_3$: The commitment $d_{\mathsf{P}}$ is computed as commitment to 0.
**Hybrid** $\mathsf{H}_4$: The ciphertext $c_{\mathsf{P}}$ is computed as $\mathsf{Enc}(\mathsf{pk}_{\mathsf{P}}, 0)$.
**Hybrid** $\mathsf{H}_5$: The pair $(\mathfrak{s}_{(1,\mathsf{Full})}, \mathfrak{p}_{(1,\mathsf{Full})})$ is computed using a uniformly sampled string, instead of the output of the $\mathsf{PRF}$.

The distance between each hybrid can be bound to a negligible function with a standard reduction to the corresponding cryptographic primitive. Note that already in hybrid 1 the witness $w$ is information-theoretically hidden from the eyes of the adversary, which guarantees zero-knowledge for the first iteration of the protocol.

To argue about zero-knowledge for subsequent executions, we need to ensure that the parameters $(\mathfrak{s}_{(1,\mathsf{Full})}, \mathfrak{p}_{(1,\mathsf{Full})})$ are correctly generated and that $\mathfrak{s}_{(1,\mathsf{Full})}$ is hidden from the eyes of the adversary. Clearly, this is the case for the second session in hybrid 5. Then we can apply the same sequence of hybrids and derive zero-knowledge for the $i$-th session by an inductive argument.

**Computationally Unique Transcripts.** We consider explicitly only the case where the prover's public parameters are generated honestly. The proof for where the verifier's pre-processing is honest can be shown with an analogous argument.

We begin by showing computationally unique transcripts for the first iteration of the protocol. Observe that the statement being proven by the two ZK protocols is uniquely determined by $x$ and by the public parameters of the scheme. Assume for the moment that $(x, c'_{\mathsf{P}}, d_{\mathsf{P}}, m_{\mathsf{P}}) \in \mathcal{L}_{\mathsf{Full}}$ and $(c'_{\mathsf{V}}, d_{\mathsf{V}}, m_{\mathsf{V}}) \in \mathcal{L}_{\mathsf{Dec}}$. Note that the corresponding witnesses consist of the tuples $(w, \mathsf{sk}_{\mathsf{P}}, \gamma_{\mathsf{P}})$ and $(\mathsf{sk}_{\mathsf{V}}, \gamma_{\mathsf{V}})$. Both pairs $(\mathsf{sk}_{\mathsf{P}}, \gamma_{\mathsf{P}})$ and $(\mathsf{sk}_{\mathsf{V}}, \gamma_{\mathsf{V}})$ are uniquely determined by the public parameters since the commitment scheme is perfectly binding. By the computationally unique transcripts of $(\mathsf{P}_{\mathsf{Dec}}, \mathsf{V}_{\mathsf{Dec}})$, it is computationally hard to find two accepting transcripts $\mathcal{T}_{\mathsf{Dec}}$ and $\mathcal{T}'_{\mathsf{Dec}}$. By the computationally unique transcripts of $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$ we have that, for a statement $x$ with unique witnesses, it is hard to output two accepting transcripts $\mathcal{T}_{\mathsf{Full}}$ and $\mathcal{T}'_{\mathsf{Full}}$. By the correctness of the FHE scheme the pair $(m_{\mathsf{P}}, m_{\mathsf{V}})$ is also uniquely determined by the public parameters of the scheme. We are left with the case where either $(x, c'_{\mathsf{P}}, d_{\mathsf{P}}, m_{\mathsf{P}}) \notin \mathcal{L}_{\mathsf{Full}}$ or $(c'_{\mathsf{V}}, d_{\mathsf{V}}, m_{\mathsf{V}}) \notin \mathcal{L}_{\mathsf{Dec}}$. In both cases the observer aborts with probability 1, since either $\Theta_{\mathsf{Full}}$ or $\Theta_{\mathsf{Dec}}$ aborts by the observer soundness of the corresponding protocol.

Clearly the same holds for the $i$-th iteration if the corresponding public parameters are honestly generated. In other words, $\mathfrak{p}_{1,\mathsf{Full}} \in \mathsf{P}^1_{\mathsf{Full}}$ and $\mathfrak{p}_{2,\mathsf{Dec}} \in \mathsf{V}^1_{\mathsf{Dec}}$. This can be shown via an inductive argument (see the proof of observer soundness) assuming the observer soundness of both $(\mathsf{P}_{\mathsf{Full}}, \mathsf{V}_{\mathsf{Full}})$ and $(\mathsf{P}_{\mathsf{Dec}}, \mathsf{V}_{\mathsf{Dec}})$.

## 6 Optimizations

In the following, we suggest concrete instantiations for our building blocks and we discuss several optimizations to improve the performance of our zero-knowledge protocol. For the perfectly binding commitment scheme with unique opening the canonical choice is the classical ElGamal commitment [23]:

On input a message $m \in \mathbb{Z}_p$, sample two random integer $(x, y) \leftarrow_\$ \mathbb{Z}_p$ and publish $c = (g^x, g^y, g^{xy} \cdot g^m)$ where $g$ is the generator of a cyclic group of prime order $p$. The scheme can be shown to be hiding under the decisional Diffie-Hellman assumption [19] and the value of $m$ is uniquely determined by $c$. In a similar way we can instantiate the one-way function with efficiently checkable range with the discrete logarithm problem, i.e., find $x$ such that $g^x = h$. Note that checking the range consists of determining whether $h$ corresponds to a valid group element, which can be checked efficiently (for most of the groups where discrete logarithm is conjectured to be hard, such as $\mathbb{Z}_p^*$). The discrete logarithm is also a suitable candidate for an average-case hard language with unique witnesses since the function is one-to-one.

The more interesting aspect is how to instantiate the HPP-NIWI proofs in conjunction with the SPS-ZK proofs, which is currently the efficiency bottleneck of our approach. On a high level, our proposal for an efficient HPP-NIWI (fully described in Section A) is the following: The (honest) pre-processing consists of the garbling of a circuit that takes as input a statement $x$ and a witness $w$ and computes $\mathcal{R}(x, w)$. Then it publishes the corresponding garbled circuit along with the commitments for each input label. The labels corresponding to the wires of $w$ are randomly permuted so that the later phase remains zero-knowledge. In the execution phase, the prover simply outputs the openings to the labels corresponding to the witness and the statement and the verifier evaluates the circuit and returns whatever the circuit returns. The important aspect of this scheme is that is *delayed input*: The witness and the statement do not need to be known during the pre-processing.

Soundness of the HPP-NIWI is guaranteed by the fact that the pre-processing phase is honest and therefore the verifier is assured that the circuit indeed computes $\mathcal{R}(x, w)$. Of course, this is not necessarily the case as the prover might be corrupted from the very beginning, which is the reason why we needed to add an SPS-ZK proof to certify that the pre-processing was indeed well-formed. However, the combination of the two building blocks leads to a circuit-inside-circuit structure that hinders the practicality of our approach: The SPS-ZK (in turn based on garbled circuits) needs to prove that the circuit of the HPP-NIWI was correctly garbled. Note that we cannot directly use the SPS-ZK as an HPP-NIWI since the protocol is not delayed input.

**Cut-and-Choose to the Rescue.** Fortunately, there is a way to combine the two instantiation of HPP-NIWI and SPS-ZK in a non-blackbox way, using the techniques of [42]. This (conceptually more involved but more efficient) approach will avoid the nested garbling structure, thereby lifting our solution to the realm of practicality. The high-level idea of the protocol is the following: In the pre-processing the prover garbles $s$ copies of the same circuit computing $\mathcal{R}(x, w)$. For the input wires corresponding to $x$, it simply commits the corresponding labels in some fixed order. For each input wire $i$ corresponding to a bit of $w$, the prover samples a random bit $b \leftarrow_\$ \{0, 1\}$ and computes $s$-many commitment sets $\{W_{i,j}, \tilde{W}_{i,j}\}_{j \in [s]}$ as

$$W_{i,j} = (\mathsf{Com}(b), \mathsf{Com}(\ell_{i,1}^b), \ldots, \mathsf{Com}(\ell_{i,s}^b))$$
$$\tilde{W}_{i,j} = (\mathsf{Com}(b \oplus 1), \mathsf{Com}(\ell_{i,1}^{b \oplus 1}), \ldots, \mathsf{Com}(\ell_{i,s}^{b \oplus 1})).$$

where $(\ell_{i,j}^0, \ell_{i,j}^1)$ is the pair of labels of the $i$-th input wire of the $j$-th circuit. The challenge of the verifier $\beta \in \{0, 1\}^{2s}$ determines which circuits and which commitment sets to open. We call these check circuits (sets, respectively) and the complementary set evaluation circuits (sets, respectively).

In the execution phase the prover (who is now aware of $x$ and $w$) reveals the opening of the following commitments:

1. All check sets corresponding to the witness wires of all check circuits.
2. All commitments in the evaluation sets for the wires corresponding to $w_i$ for all evaluation circuits.
3. All commitments to the statement wires for the check circuits.
4. All commitments for the wires corresponding to $x_i$ for all evaluation circuits.
5. The first commitment (for the bit $b$) to all check sets.

The verifier accepts if all of the openings are consistent with the honest pre-processing and if all evaluation circuits return 1. Note that the protocol is delayed-input and therefore it is suitable for our purposes. Soundness is ensured by the fact that a random half of the circuits are checked and therefore the cheating prover will be caught with all but negligible probability. The uniqueness of the second message is due to the fact that the decommitments are unique and that the check sets ensure that the value of $w$ is consistent across all evaluation circuits. Finally, it is easy to see that the protocol is simulatable in super-polynomial time by simply guessing the challenge $\beta$ ahead of time. We refer the reader to [42] for a detailed analysis.

The protocol outlined above combines the features of an HPP-NIWI and SPS-ZK and eliminates the nested use of garbled circuits. Combined with the other building blocks, our final SF-ZK protocol can be built using only Yao's garbled circuit and discrete-logarithm commitments, which are two well-studied and efficient primitives.

## 7 Impossibility of Fully Malicious Preprocessing

Given that we can only instantiate steganography-free zero-knowledge argument systems in a setting where at least one party acts honestly in the preprocessing phase, it is natural to ask whether this trust anchor is necessary. I.e., is it possible to achieve steganography-freeness for useful languages when both parties are malicious during the pre-processing.

For this we consider a strengthened version of Definition 8. Recall that selective observer soundness and computationally unique transcripts in Definition 8 use a notion of admissible pairs of preprocessing algorithms, where at least one algorithm needs to be honest. I.e., a pair $(\tilde{\mathsf{P}}^1, \tilde{\mathsf{V}}^1)$ is considered admissible if both algorithms are $\mathsf{PPT}$ and it holds that $\tilde{\mathsf{P}}^1(w,x) = \mathsf{P}^1(1^n)$ or $\tilde{\mathsf{V}}^1(x) = \mathsf{V}^1(1^n)$. We essentially relax this condition here and allow for any pair of $\mathsf{PPT}$ algorithms to be admissible. Formally this is specified in the following definitions.

**Definition 11 (Non-Adaptive Observer Soundness with Fully Malicious Preprocessing).** *For all $x \notin \mathcal{L}$, for all $\mathsf{PPT}$ algorithms $\mathsf{P}^*$ and $\mathsf{V}^*$ it holds that*

$$\Pr\left[\begin{matrix}(s_1,p_1) \leftarrow \mathsf{P}^*(x), (s_2,p_2) \leftarrow \mathsf{V}^*(x), \\ \mathcal{T} := \langle \mathsf{P}^*(s_1,p_2,x,w), \mathsf{V}^*(s_2,p_1,x)\rangle\end{matrix} : \Theta(p_1,p_2,\mathcal{T},x) = 1\right] \leq \mathsf{negl}(n)$$

**Definition 12 (Computationally Unique Transcripts with Fully Malicious Preprocessing).** *For all $x \in \mathcal{L}$ such that there exists a unique $w$ such that $\mathcal{R}(x,w) = 1$, for all $\mathsf{PPT}$ algorithms $\left(\mathsf{P}^*, \mathsf{V}^*, \hat{\mathsf{P}}^*, \hat{\mathsf{V}}^*\right)$ it holds that*

$$\Pr\left[\begin{matrix}(s_1,p_1) \leftarrow \mathsf{P}^*(w,x), (s_2,p_2) \leftarrow \mathsf{V}^*(x), \\ \mathcal{T}_1 := \langle \mathsf{P}^*(s_1,p_2,x,w), \mathsf{V}^*(s_2,p_1,x)\rangle, \\ \mathcal{T}_2 := \left\langle \hat{\mathsf{P}}^*(s_1,p_2,x,w), \hat{\mathsf{V}}^*(s_2,p_1,x)\right\rangle\end{matrix} : \begin{matrix}\Theta(p_1,p_2,\mathcal{T}_1,x) = 1 \wedge \\ \Theta(p_1,p_2,\mathcal{T}_2,x) = 1 \wedge \\ \mathcal{T}_1 \neq \mathcal{T}_2\end{matrix}\right] \leq \mathsf{negl}(n)$$

We will prove that this stronger notion is not achievable for languages outside of BPP. The idea behind the proof is that any steganography-free zero-knowledge argument that is also observer sound with fully malicious preprocessing can actually be compressed into a non-interactive argument for the same language. In the compressed argument, the prover acts as the verifier in the protocol and simply outputs a preprocessing and a transcript. Observer completeness and observer soundness with fully malicious preprocessing guarantee that the proof nevertheless remains verifiable and sound. However, the resulting argument is a non-interactive zero-knowledge argument in the standard model (i.e. without a common reference string) which exist only for languages in BPP [30].

Formally we prove the following theorem.

**Theorem 7.** *Let $\Pi$ be a steganography free argument for language $\mathcal{L}$ that is also observer sound with fully malicious preprocessing. Then $\mathcal{L} \in BPP$.*

*Proof.* We use $\Pi$ to construct a non-interactive argument $\mathsf{NIZK} = (\overline{\mathsf{P}}, \overline{\mathsf{V}})$ as depicted in Figure 4.

**Lemma 3.** *If $\Pi$ is zero knowledge, observer complete, and observer sound with fully malicious preprocessing, then $\mathsf{NIZK}$ is a complete, sound, and zero knowledge non-interactive argument.*

*Proof.* We prove completeness, soundness, and zero-knowledge separately.

**Completeness.** The completeness of $\mathsf{NIZK}$ follows directly from observer completeness of $\Pi$. The output of the honest prover $\overline{\mathsf{P}}$ is an honest preprocessing and an honestly generated transcript of the execution phase. Since $\overline{\mathsf{V}}$ simply invokes the observer, observer completeness immediately implies that the verifier outputs 1 with probability $1 - \mathsf{negl}(n)$.

| $\overline{\mathsf{P}}(x,w)$ | $\overline{\mathsf{V}}(x,\pi)$ |
|---|---|
| $(s_1,p_1) \leftarrow \mathsf{P}^1(1^n)$ | $\pi = (p_1,p_2,\mathcal{T})$ |
| $(s_2,p_2) \leftarrow \mathsf{V}^1(1^n)$ | **return** $\Theta(p_1,p_2,\mathcal{T},x)$ |
| $\mathcal{T} := \langle \mathsf{P}^2(s_1,p_2,x,w), \mathsf{V}^2(s_2,p_1,x)\rangle$ | |
| **return** $\pi := (p_1,p_2,\mathcal{T})$ | |

**Fig. 4.** The non-interactive argument system $\mathsf{NIZK}$ constructed by compressing $\Pi$.

**Zero Knowledge.** To prove that $\mathsf{NIZK}$ is zero-knowledge we specify a simulator $\overline{\mathsf{Sim}}$. This simulator will work by running the ZK-simulator $\mathsf{Sim}$ of the underlying $\Pi$ against the honest verifier. I.e., given $x \in \mathcal{L}$, it runs $(s_1,p_1) \leftarrow \mathsf{Sim}(1^n)$ to get the prover's simulated preprocessing and $(s_2,p_2) \leftarrow \mathsf{V}^1(1^n)$ to get an honest verifier preprocessing. The simulator then executes $\mathcal{T} := \langle \mathsf{Sim}(x,p_2), \mathsf{V}^2(x,s_2,p_1)\rangle$, where $\mathcal{T}$ refers to the transcript as seen by $\mathsf{V}$, i.e., without any of the potential rewindings $\mathsf{Sim}$ may have used. $\overline{\mathsf{Sim}}$ then outputs $\pi := (p_1,p_2,\mathcal{T})$.

Now assume towards contradiction that there exists a $\mathsf{PPT}$ distinguisher $\mathcal{D}$, such that

$$\left|\Pr\left[\pi \leftarrow \overline{\mathsf{P}}(x,w) : \mathcal{D}(x,\pi)=1\right] - \Pr\left[\pi \leftarrow \overline{\mathsf{Sim}}(x) : \mathcal{D}(x,\pi)=1\right]\right| \geq \frac{1}{\mathsf{poly}\,(n)}$$

We then construct a malicious verifier $\mathsf{V}^*$ against the zero-knowledge property of $\Pi$ as follows. In both the preprocessing phase and the execution phase $\mathsf{V}^*$ behaves like an honest verifier, except that at the end of the execution phase it outputs $p_1, p_2$ as well as the transcript $\mathcal{T}$ consisting of all messages in its view.

It is easy to see that whenever $\mathsf{V}^*$ on input $x$ communicates with an honest prover $\mathsf{P}$ with input $x,w$, its output is distributed identically to the output of $\overline{\mathsf{P}}$ on input $x,w$. I.e., it holds that

$$\begin{aligned} &\Pr\left[\begin{matrix}(s_1,p_1) \leftarrow \mathsf{P}^1(1^n),\\ (s_2,p_2) \leftarrow \mathsf{V}^*(x)\end{matrix} : \mathcal{D}(\langle \mathsf{P}^2(x,w,p_2), \mathsf{V}^*(x,p_1)\rangle) = 1\right] \\ =&\Pr\left[\pi \leftarrow \overline{\mathsf{P}}(x,w) : \mathcal{D}(x,\pi)=1\right] \end{aligned} \tag{5}$$

Similarly, whenever $\mathsf{V}^*$ on input $x$ communicates with an the simulator $\mathsf{Sim}$ with input $x$, its output is distributed identically to the output of $\overline{\mathsf{Sim}}$ on input $x$. I.e., it holds that

$$\begin{aligned} &\Pr\left[\begin{matrix}(s_1,p_1) \leftarrow \mathsf{Sim}(1^n),\\ (s_2,p_2) \leftarrow \mathsf{V}^*(x)\end{matrix} : \mathcal{D}(\langle \mathsf{Sim}(x,p_2), \mathsf{V}^*(x,p_1)\rangle) = 1\right] \\ =&\Pr\left[\pi \leftarrow \overline{\mathsf{Sim}}(x) : \mathcal{D}(x,\pi)=1\right] \end{aligned} \tag{6}$$

Combining Equation 5 and Equation 6 we can thus conclude that

$$\begin{aligned} &\left|\begin{matrix} \Pr\left[\begin{matrix}(s_1,p_1) \leftarrow \mathsf{P}^1(1^n),\\ (s_2,p_2) \leftarrow \mathsf{V}^*(x)\end{matrix} : \mathcal{D}(\langle \mathsf{P}^2(x,w,p_2), \mathsf{V}^*(x,p_1)\rangle) = 1\right] \\ -\Pr\left[\begin{matrix}(s_1,p_1) \leftarrow \mathsf{Sim}(1^n),\\ (s_2,p_2) \leftarrow \mathsf{V}^*(x)\end{matrix} : \mathcal{D}(\langle \mathsf{Sim}(x,p_2), \mathsf{V}^*(x,p_1)\rangle) = 1\right] \end{matrix}\right| \\ =&\left|\Pr\left[\pi \leftarrow \overline{\mathsf{P}}(x,w) : \mathcal{D}(x,\pi)=1\right] - \Pr\left[\pi \leftarrow \overline{\mathsf{Sim}}(x) : \mathcal{D}(x,\pi)=1\right]\right| \\ \geq&\frac{1}{\mathsf{poly}\,(n)} \end{aligned}$$

which would contradict that $\Pi$ is zero-knowledge. Therefore a distinguisher as assumed above cannot exist and the output of $\overline{\mathsf{Sim}}$ must be indistinguishable from the output of an honest prover. Therefore, $\mathsf{NIZK}$ is zero-knowledge.

**Soundness.** It remains to show that $\mathsf{NIZK}$ is also sound. This is where $\Pi$'s observer soundness with fully malicious preprocessing comes in to play.

Assume towards contradiction, that there exists an $x^* \notin \mathcal{L}$ and a malicious $\mathsf{PPT}$ prover $\overline{\mathsf{P}}^*$, such that

$$\Pr\left[\pi \leftarrow \overline{\mathsf{P}}^*(1^n) : \overline{\mathsf{V}}(x^*, \pi) = 1\right] \geq \frac{1}{\mathsf{poly}(n)}$$

Let's assume without loss of generality, that $\overline{\mathsf{P}}^*$ is deterministic by fixing its random coins such that the above probability is maximized. We denote the output of deterministic $\overline{\mathsf{P}}^*$ as $\pi^* = (p_1^*, p_2^*, \mathcal{T}^*)$.

We construct malicious prover $\mathsf{P}^*$ and verifier $\mathsf{V}^*$ for $\Pi$ as follows. In the preprocessing phase $\mathsf{P}^*$ outputs $(\bot, p_1^*)$ and $\mathsf{V}^*$ outputs $(\bot, p_2^*)$. In the execution phase $\mathsf{P}^*$ and $\mathsf{V}^*$ then simply send the messages specified in $\mathcal{T}^*$ in the appropriate order. It then follows that

$$\Pr\left[\begin{array}{l}(s_1, p_1) \leftarrow \mathsf{P}^*(x^*), (s_2, p_2) \leftarrow \mathsf{V}^*(x^*), \\ \mathcal{T} := \langle \mathsf{P}^*(s_1, p_2, x, w), \mathsf{V}^*(s_2, p_1, x)\rangle\end{array} : \Theta(p_1, p_2, \mathcal{T}, x^*) = 1\right] \tag{7}$$

$$= \Pr\left[\Theta(p_1^*, p_2^*, \mathcal{T}^*, x^*) = 1\right] \tag{8}$$

$$= \Pr\left[\overline{\mathsf{V}}(x^*, \pi^*) = 1\right] \geq \frac{1}{\mathsf{poly}(n)}, \tag{9}$$

where the first equality follows by construction of $\mathsf{P}^*$ and $\mathsf{V}^*$ and the second equality follows by construction of $\overline{\mathsf{V}}$. Since this would contradict $\Pi$'s observer soundness with fully malicious preprocessing, $\mathsf{P}^*$ cannot exist and $\mathsf{NIZK}$ must be sound.

**Lemma 4.** *If* $\mathsf{NIZK}$ *is complete, sound, and zero-knowledge then* $\mathcal{L} \in$ *BPP.*

*Proof.* The lemma follows immediately from the seminal result of Goldreich and Oren [30], that "one-step zero-knowledge proofs" only exist for languages in BPP. However, we shortly recall the idea of the proof here.

We first note that for any $x \in \mathcal{L}$ completeness of the NIZK implies that

$$\Pr\left[\pi \leftarrow \overline{\mathsf{P}}(x, w) : \overline{\mathsf{V}}(x, \pi) = 1\right] \geq 1 - \mathsf{negl}(n).$$

Further, since the NIZK is also zero-knowledge and simulated proofs are thus indistinguishable from real proofs, it follows that

$$\Pr\left[\pi \leftarrow \overline{\mathsf{Sim}}(x) : \overline{\mathsf{V}}(x, \pi) = 1\right] \geq 1 - \mathsf{negl}(n). \tag{10}$$

For any $x \notin \mathcal{L}$ on the other hand, soundness guarantees that for all $\mathsf{PPT}$ algorithms $\overline{\mathsf{P}}^*$ it holds that

$$\Pr\left[\pi \leftarrow \overline{\mathsf{P}}^*(x) : \overline{\mathsf{V}}(x, \pi) = 1\right] \leq \mathsf{negl}(n).$$

And since $\overline{\mathsf{Sim}}$ is in particular a $\mathsf{PPT}$ algorithm, it thus follows that

$$\Pr\left[\pi \leftarrow \overline{\mathsf{Sim}}(x) : \overline{\mathsf{V}}(x, \pi) = 1\right] \leq \mathsf{negl}(n). \tag{11}$$

We can thus construct a machine $\mathsf{M}_{\mathcal{L}}$ that takes as input a word $x$, computes $\pi \leftarrow \overline{\mathsf{Sim}}(x)$ and accepts if and only if $\overline{\mathsf{V}}(x, \pi) = 1$. It follows from Equation 10 and Equation 11 that $\mathsf{M}_{\mathcal{L}}$ accepts $x \in \mathcal{L}$ with probability at least $1 - \mathsf{negl}(n)$ and accepts $x \notin \mathcal{L}$ with probability at most $\mathsf{negl}(n)$. It thus follows that $\mathcal{L} \in$ BPP.

Theorem 7 follows immediately from the two lemmas.

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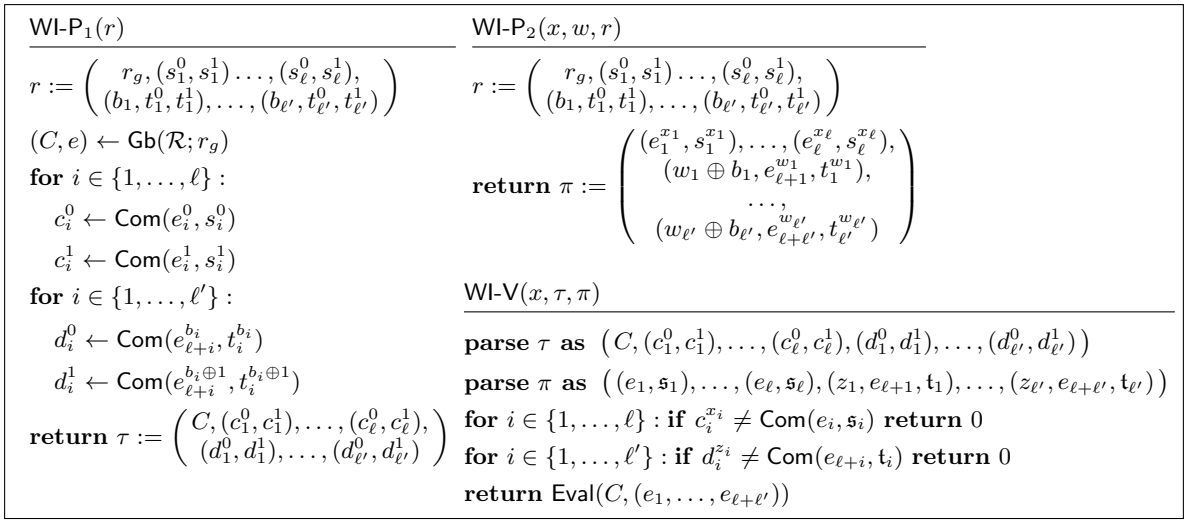


**Fig. 5.** The HPP-NIWI protocol.

# A HPP-NIWI Construction

An HPP-NIWI can be constructed based on any projective garbling scheme [10]. Let $\mathsf{GC} := (\mathsf{Gb}, \mathsf{Eval})$ be a projective garbling scheme and let $\mathsf{Com}$ be a perfectly binding commitment scheme with unique openings. Let $\ell$ be an upper bound on the size of the statement and let $\ell'$ be an upper bound on the size of the witness. The HPP-NIWI protocol is constructed in Figure 5.

**Theorem 8 (Soundness).** *Let* $(\mathsf{Gb}, \mathsf{Eval})$ *be a projective garbling scheme and let* $\mathsf{Com}$ *be a perfectly binding commitment scheme, then the HPP-NIWI protocol in Figure 5 is sound.*

*Proof.* Let WI-P$^*$ be a machine that output a tuple $(x^*, \tau^*, \pi^*, r^*)$ such that $x^* \notin \mathcal{L}$, WI-P$_1(r^*) = \tau^*$, and WI-V$(x^*, \tau^*, \pi^*) = 1$. Since $\tau$ has to be honestly generated then it is of the form

$$(C, (c_1^0, c_1^1), \ldots, (c_\ell^0, c_\ell^1), (d_1^0, d_1^1), \ldots, (d_{\ell'}^0, c_{\ell'}^1)),$$

where $C$ is a garbled circuits and the rest of the pairs are commitments to the labels of $C$. By correctness of the garbling scheme and since $x^* \notin \mathcal{L}$, there exists no well-formed input encoding $e$ such that $\mathsf{Eval}(C, e) = 1$. This implies that $\pi^*$ must contain at least one pair $(e_i^*, \mathfrak{s}_i^*)$ (or a pair $(e_i^*, \mathfrak{t}_i^*)$) such that $e_i^* \neq e_i^0$ and $e_i^* \neq e_i^1$, but $\mathsf{Com}(e_i^*, \mathfrak{s}_i^*) = c_i$ (or $\mathsf{Com}(e_i^*, \mathfrak{t}_i^*) = d_i$). This is a contradiction to the perfect binding property of the commitment scheme.

**Theorem 9 (Witness Indistinguishability).** *Let* $(\mathsf{Gb}, \mathsf{Eval})$ *be a private projective garbling scheme and let* $\mathsf{Com}$ *be a computationally hiding commitment scheme, then the HPP-NIWI protocol in Figure 5 is witness indistinguishable.*

*Proof.* We modify the witness indistinguishability game in a sequence of hybrids and then we argue about the proximity of each pair of neighbouring experiment

**Hybrid** $\mathsf{H}_1$: Defined as the honest execution.
**Hybrid** $\mathsf{H}_2$: All commitments for the labels that do not correspond to $(x, w)$ are computed as commitments to 0.
**Hybrid** $\mathsf{H}_3$: The garbled circuit and the input encoding are computed using the simulator $(C, (e_1, \ldots, e_{\ell+\ell'})) \leftarrow \mathsf{Sim}(1)$.
**Hybrid** $\mathsf{H}_4$: For each $i \in \{1, \ldots, \ell'\}$ a coin $\mathfrak{z}_i$ is flipped and the label $e_i$ is planted in the commitment $d_i^{\mathfrak{z}_i}$. Then each $z_i$ is set to $\mathfrak{z}_i$.

We will show that for each $i \in \{1, 2, 3\}$ the value of $\delta_i$ is negligible, where $\delta_i$ is defined as

$$\delta_i := \left| \Pr\left[(\tau, \pi) \leftarrow \mathsf{H}_i : \mathcal{D}(\tau, \pi) = 1\right] - \Pr\left[(\tau, \pi) \leftarrow \mathsf{H}_{i+1}\right] : \mathcal{D}(\tau, \pi) = 1 \right|,$$

where $\mathsf{H}_i$ is defined to be the execution of hybrid $i$. Since the commitment is computationally hiding, a standard hybrid argument can be used to bound $\delta_1$ to a negligible value. Since $\mathsf{H}_2$ and $\mathsf{H}_3$ differ only in how the tuple $(C, e)$ is computed, if $\delta_2$ was non-negligible, then $\mathcal{D}$ would be a successful distinguisher against the privacy of the projective garbling scheme $(\mathsf{Gb}, \mathsf{Eval})$. Therefore, we can bound $\delta_2$ to a negligible value as well. Finally, we observe that the changes between $\mathsf{H}_3$ and $\mathsf{H}_4$ are only syntactical since the string $b_1, \ldots, b_{\ell'}$ is uniformly sampled and information-theoretically hides the witness. It follows that $\delta_3 = 0$.

At this point, we observe that the output of $\mathsf{H}_4$ carries no information about the witness $w$. It follows that no distinguisher can tell whether $w_1$ or $w_2$ was used. This concludes our proof.

**Theorem 10 (Uniqueness).** *Let* $(\mathsf{Gb}, \mathsf{Eval})$ *be a projective garbling scheme and let* $\mathsf{Com}$ *be a perfectly binding commitment scheme, then the HPP-NIWI protocol in Figure 5 has unique proofs.*

*Proof.* By the correctness of the garbling scheme $(\mathsf{Gb}, \mathsf{Eval})$, for all strings $v \in \{0,1\}^{\ell'}$ such that $\mathcal{R}(x, v) \neq 1$ it holds that $\mathsf{Eval}(C, (e_1^{v_1}, \ldots, e_{\ell'}^{v_{\ell'}})) \neq 1$. It follows that $\mathsf{Eval}(C, (e_1^{w_1}, \ldots, e_{\ell'}^{w_{\ell'}})) = 1$ if and only if $\mathcal{R}(x, w) = 1$, which implies that for each string $w$ there exists exactly one combination of labels that causes the verifier to accept. Since the commitment scheme is perfectly binding, for each commitment $c_i$ (or $d_i$) there exists exactly one valid opening $\mathfrak{s}_i$ (or $\mathfrak{t}_i$). This also implies that once the labels and the openings are fixed, then the position bits $(z_i, \ldots, z_{\ell'})$ are uniquely determined. It follows that the string $\pi$ is uniquely determined by $w$ and $\tau$.

**Theorem 11 (Extractability).** *Let* $(\mathsf{Gb}, \mathsf{Eval})$ *be a projective garbling scheme and let* $\mathsf{Com}$ *be a perfectly binding commitment scheme, then the HPP-NIWI protocol in Figure 5 is extractable.*

*Proof.* The extractor parses $r$ as

$$\big(r_g, (s_1^0, s_1^1) \ldots, (s_\ell^0, s_\ell^1), (b_1, t_1^0, t_1^1), \ldots, (b_{\ell'}, t_{\ell'}^0, t_{\ell'}^1)\big)$$

and $\pi$ as

$$((e_1, \mathfrak{s}_1), \ldots, (e_\ell, \mathfrak{s}_\ell), (z_1, e_{\ell+1}, \mathfrak{t}_1), \ldots, (z_{\ell'}, e_{\ell+\ell'}, \mathfrak{t}_{\ell'}))$$

and returns $b_1 \oplus z_1 \| \ldots \| b_{\ell'} \oplus z_{\ell'}$. Note that $\tau$ is honestly generated and, as argued in Theorem 10, $\pi$ is uniquely determined by $(\tau, w)$. It follows that $\pi$ must be well-formed and therefore the extractor is successful with probability 1.