Amplification, Redundancy, and the Quantum Chernoff Information

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Amplification was regarded, since the early days of quantum theory, as a mysterious ingredient that endows quantum microstates with macroscopic consequences, key to the "collapse of the wave packet", and a way to avoid embarrassing problems exemplified by Schrdinger's cat. Such a bridge between the quantum microworld and the classical world of our experience was postulated *ad hoc* in the Copenhagen interpretation. Quantum Darwinism views amplification as replication, in many copies, of the information about quantum states. We show that such amplification is a natural consequence of a broad class of models of decoherence, including the photon environment we use to obtain most of our information. This leads to objective reality via the presence of robust and widely accessible records of selected quantum states. The resulting redundancy (the number of copies deposited in the environment) follows from the quantum Chernoff information that quantifies the information transmitted by a typical elementary subsystem of the environment.

Building on the theory of decoherence [1–3], quantum Darwinism is a framework to go beyond the Copenhagen interpretation and "bridge" the quantum-classical divide [4]. It recognizes that the environment acts as a communication channel for information about a system of interest, $\mathcal{S}$. Observers acquire information indirectly by intercepting a fragment $\mathcal{F}$ of the environment $\mathcal{E}$, such as scattered photons, as happens in everyday life (see Fig. 1). This is possible because correlations are created between $\mathcal{S}$ and $\mathcal{E}$ when they interact. They can be quantified by the quantum mutual information $I\left(\mathcal{S}:\mathcal{F}\right) = H_{\mathcal{S}} + H_{\mathcal{F}} - H_{\mathcal{SF}}$, where $H_{\mathcal{A}} = -\mathrm{tr}\rho_{\mathcal{A}} \log_2 \rho_{\mathcal{A}}$ are the von Neumann entropies. Correlations between elusive quantum states, when amplified, dependably lead to "objective classical reality". In this Letter, we prove that a broad class of photon and photonlike environments always amplify information.

The quantum mutual information is naturally divided into classical and quantum contributions [5] – the Holevo quantity [6, 7] and quantum discord [8–10], respectively. Here, we will focus on the Holevo quantity and hence the information accessible via $\mathcal{E}$ about the system $\mathcal{S}$ – about its pointer observable [11] $\hat{\Pi}_{\mathcal{S}} = \sum_{\hat{s}} \pi_{\hat{s}} |\hat{s}\rangle\langle\hat{s}|$,

$$\chi\left(\hat{\Pi}_{\mathcal{S}}:\mathcal{F}\right) = H\left(\sum_{\hat{s}} p_{\hat{s}} \rho_{\mathcal{F}|\hat{s}}\right) - \sum_{\hat{s}} p_{\hat{s}} H\left(\rho_{\mathcal{F}|\hat{s}}\right). \quad (1)$$

The Holevo quantity upper bounds the classical information (information about the pointer states [5, 12]) transmittable by a quantum channel (here, the environment), as well as lower bounds the quantum mutual information, $I\left(\mathcal{S}:\mathcal{F}\right) \geq \chi\left(\hat{\Pi}_{\mathcal{S}}:\mathcal{F}\right)$. In this expression, $\hat{s} = 1, \ldots, D_{\mathcal{S}}$ labels the pointer states, $p_{\hat{s}}$ are their probabilities, and $\rho_{\mathcal{F}|\hat{s}} = \langle\hat{s}|\rho_{\mathcal{SF}}|\hat{s}\rangle/p_{\hat{s}}$ are the "messages" about $\mathcal{S}$ transmitted by $\mathcal{F}$ – the fragment state conditioned on the system's pointer state $\hat{s}$. We will focus on the case where $\mathcal{S}$ is two dimensional, although the overall conclusions hold for higher dimensions.

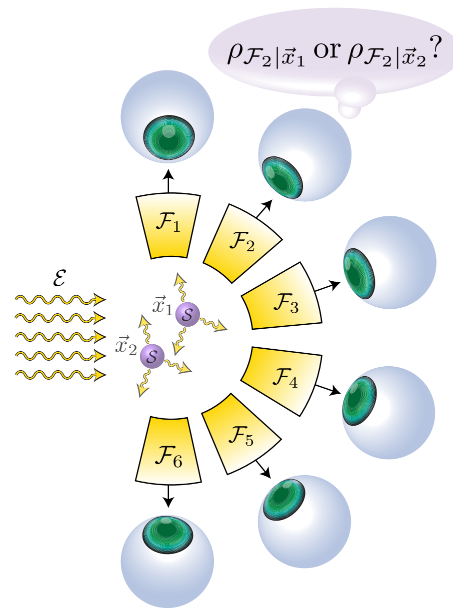

Figure 1: Quantum Darwinism, photons, and the emergence of objective classical reality. A quantum system, $\mathcal{S}$, initially in a nonlocal superposition, is illuminated by the environment, $\mathcal{E}$, composed of many distinct subsystems (photons) that can be lumped into fragments $\mathcal{F}$. While $\mathcal{E}$ decoheres $\mathcal{S}$, it acquires many copies of information about $\mathcal{S}$ that become available to observers who can then independently infer the state of the preferred (pointer) state of the system without perturbing $\mathcal{S}$ by direct measurements. This redundant imprinting of records is responsible for the consensus between observers that is essential for the emergence of "objective classical reality" in our quantum Universe. For the familiar photon environment, the redundancy (which quantifies amplification) can be enormous: A dust grain 1 μm across exposed to sunlight for just 1 μs will have its location (to an accuracy of 1 μm) recorded about 10^8 times in the scattered photons [13].

For information to be objective – and therefore for the quantum world to conform to our everyday experience – many observers should be able to access it independently

[12, 14]. The missing information about $\mathcal{S}$ is quantified by its entropy $H_{\mathcal{S}}$, and this information must be redundantly proliferated into the world for it to be objective. In other words, each observer should only need a small fragment of the environment to retrieve it. This will allow many observers to independently determine the state of the system, and reach consensus about it, accounting for the emergence of objective classical reality in the quantum Universe. More precisely, the number of fragments of the environment that contain sufficient information about $\mathcal{S}$ can be deduced starting from the condition

$$\langle \chi \left(\hat{\Pi}_{\mathcal{S}} : \mathcal{F} \right) \rangle_{\sharp\mathcal{F}_\delta} \cong (1-\delta)\, H_{\mathcal{S}}, \tag{2}$$

where $\langle \cdot \rangle_{\sharp\mathcal{F}_\delta}$ designates an average over fragments of size $^\sharp\mathcal{F}_\delta$ and $H_{\mathcal{S}} = H(\hat{\Pi}_{\mathcal{S}})$, i.e., the entropy of the system is the entropy of the pointer observable when the system is decohered. The fragment size $^\sharp\mathcal{F}_\delta$ is the number of subsystems of the environment (e.g., the number of scattered photons or the number of two level systems) needed for an observer to acquire $(1-\delta)\, H_{\mathcal{S}}$ bits of information, on average, about $\mathcal{S}$. The information deficit, δ, is the information observers can forgo, e.g., observers may be satisfied with 90% ($\delta = 10^{-1}$) of the missing information. The number of copies proliferated into the environment defines the redundancy (the "gain", the figure of merit for amplification) via

$$R_\delta = \frac{^\sharp\mathcal{E}}{^\sharp\mathcal{F}_\delta}, \tag{3}$$

where $^\sharp\mathcal{E}$ is the size the environment.

When environments select, but do not perturb, a definite pointer observable of a system we shall say that $\mathcal{E}$ *purely decoheres* $\mathcal{S}$. These situations are characterized by the Hamiltonians

$$\mathbf{H} = \mathbf{H}_{\mathcal{S}} + \hat{\Pi}_{\mathcal{S}} \sum_{k=1}^{\sharp\mathcal{E}} \Upsilon_k + \sum_{k=1}^{\sharp\mathcal{E}} \Omega_k \tag{4}$$

with $\left[\hat{\Pi}_{\mathcal{S}}, \mathbf{H}_{\mathcal{S}}\right] = 0$ and initial states

$$\rho(0) = \rho_{\mathcal{S}}(0) \otimes \left[\bigotimes_{k=1}^{\sharp\mathcal{E}} \rho_k(0) \right], \tag{5}$$

where k specifies an environment subsystem [32][15, 16]. In this scenario, no transitions are generated between the pointer states $\hat{s}$ (the eigenstates of $\hat{\Pi}_{\mathcal{S}}$ [2, 11]). The system can still interact with $^\sharp\mathcal{E}$ independent environment subsystems with arbitrary, and potentially different, interaction operators Υ_k and self-Hamiltonians Ω_k.

The Hamiltonian, Eq. (4), is exact in the case of some central spin models [2, 11] and can be regarded as a limiting form of one where $\left[\mathbf{H}_{\mathcal{S}}, \hat{\Pi}_{\mathcal{S}}\right] \neq 0$ but the system's evolution (through $\mathbf{H}_{\mathcal{S}}$) occurs slowly on the time scale it interacts with the environment. Such a condition is broadly true in our everyday world, as objects are rapidly decohered by collisions with air molecules and/or photons [3, 17]. Moreover, independence of the environment subsystems is satisfied essentially exactly for the photon environment, and approximately when the relevant time scales are much faster than the mixing time of the environment [18]. We assume independence as a simplification, as it is thought to be approximately necessary for preserving the redundancy of information. In the following, we will prove that it is sufficient.

To estimate R_δ, we will apply three inequalities and take $^\sharp\mathcal{F}_\delta \to \infty$. The first inequality is Fano's [7, 19] for $D_{\mathcal{S}} = 2$, which gives the lower bound

$$\chi \left(\hat{\Pi}_{\mathcal{S}} : \mathcal{F} \right) \geq H_{\mathcal{S}} - H(P_e), \tag{6}$$

where P_e, a function of $\mathcal{F}$, is the error probability to distinguish the conditional states $\rho_{\mathcal{F}|\hat{s}}$. For practical purposes, one could easily substitute the right-hand side in Eq. (6) in the definition for redundancy. However, retaining Eq. (2) in the definition of redundancy will result in the Fano inequality leading to a lower bound to R_δ. The second inequality is that established in Ref. [20],

$$\mathrm{tr}\left[A^c B^{1-c}\right] \geq \mathrm{tr}\left[A + B - |A - B|\right]/2 \tag{7}$$

for two positive operators A and B and $0 \leq c \leq 1$. This inequality was used to prove one side of the quantum Chernoff bound (QCB) [20–22], which generalizes the classical Chernoff bound to sources of independent and identically distributed (i.i.d.) *quantum* states. Using Eq. (7), we can upper bound the optimal error probability – from the Helstrom measurement [23] – for distinguishing the $D_{\mathcal{S}} = 2$ states generated on the fragment as

$$P_e \leq P_e^\star = p_1^c p_2^{1-c} \prod_{k \in \mathcal{F}} \mathrm{tr}\left[\rho_{k|1}^c\, \rho_{k|2}^{1-c}\right], \tag{8}$$

where $\rho_{k|\hat{s}}$ are the subsystem's state conditioned on the $\hat{s}$ pointer state of $\mathcal{S}$. Here, the conditional subsystem states are independent, but not identically distributed (i.e., they are not i.i.d.). Using $P_e^\star$ in Eq. (6) will further lower bound the accessible information, and thus further lower bound R_δ. The third inequality is $H(P_e^\star) \leq P_e^\star/\ln 2 - P_e^\star \log_2 P_e^\star$ [33].

We want to determine the relationship between $^\sharp\mathcal{F}_\delta$ and δ that follows from Eq. (2), which requires averaging over fragments of the same size. In principle, this could be difficult if one attempts to optimize the bound in Eq. (8) by minimizing over c, as c can depend on $\mathcal{F}$. There are important cases where the optimum c is independent of $\mathcal{F}$, such as the photon environment below, spin-1/2 environments, and environments with a pure initial state. Moreover, for the purposes of bounds one does not have to do any minimization, as one can take any c, e.g., $c = 1/2$. Hereon, we will take c as a constant. Averaging Eq.

(8) over fragments of size $^\sharp\mathcal{F}_\delta$, then taking the logarithm and limit $^\sharp\mathcal{F}_\delta \to \infty$ [34], one obtains

$$-\lim_{^\sharp\mathcal{F}_\delta\to\infty}\frac{1}{^\sharp\mathcal{F}_\delta}\ln\langle P_e\rangle \geq -\ln\langle \mathrm{tr}\left[\rho_{k|1}^c\,\rho_{k|2}^{1-c}\right]\rangle_{k\in\mathcal{E}} \equiv \bar{\xi}_{\mathrm{QCB}} \tag{9}$$

We have introduced a "typical" Chernoff information $\bar{\xi}_{\mathrm{QCB}}$, which is averaged over all subsystems k in $\mathcal{E}$ [35]. Now we need to do the same averaging and limit in order to connect $^\sharp\mathcal{F}_\delta$ and δ. Using Eq. (2) with the three inequalities, we have

$$\delta H_{\mathcal{S}} \leq \langle P_e^\star/\ln 2 - P_e^\star \log_2 P_e^\star\rangle_{^\sharp\mathcal{F}_\delta}. \tag{10}$$

When $^\sharp\mathcal{E}\to\infty$, one obtains for the averaging $\langle P_e^\star/\ln 2 - P_e^\star \log_2 P_e^\star\rangle_{^\sharp\mathcal{F}_\delta} = g\left(^\sharp\mathcal{F}_\delta\right)\exp\left[-\bar{\xi}_{\mathrm{QCB}}{}^\sharp\mathcal{F}_\delta\right]$, where $g\left(^\sharp\mathcal{F}_\delta\right)$ is a function with the property $\lim_{^\sharp\mathcal{F}_\delta\to\infty}\left[\ln g\left(^\sharp\mathcal{F}_\delta\right)\right]/{}^\sharp\mathcal{F}_\delta = 0$. Taking the logarithm and the limit $^\sharp\mathcal{F}_\delta\to\infty$ yields

$$r \geq \bar{\xi}_{\mathrm{QCB}}, \tag{11}$$

where $r = \lim_{^\sharp\mathcal{F}_\delta\to\infty,{}^\sharp\mathcal{E}\to\infty} R_\delta \ln(1/\delta)/{}^\sharp\mathcal{E}$ is a measure of the asymptotic efficiency of the amplification. In the limiting process, the extrinsic scales $^\sharp\mathcal{E}$ and δ have been removed from R_δ, and one is left with r, an intrinsic property of the model. This lower bound immediately establishes that decoherence processes given by Eqs. (4) and (5) always redundantly proliferate information.

In addition to a lower bound for r, we can also find an upper bound in many cases. Here, we show the result for i.i.d. states and $p_1 = p_2$. This makes use of the upper bound, $\chi\left(\hat{\Pi}_{\mathcal{S}}:\mathcal{F}\right) \leq H\left(\left[1-\sqrt{F\left(\rho_{\mathcal{F}|1},\rho_{\mathcal{F}|2}\right)}\right]/2\right)$, where F is the fidelity [24]. This is further upper bounded by $H\left(\left[1-2P_e\right]/2\right)$, which yields [36]

$$r \leq 2\bar{\xi}_{\mathrm{QCB}}. \tag{12}$$

This shows that the Chernoff information is the same as r up to a factor of 2. In the examples we have calculated (including the photon environment below and spin environments), it is the measure of efficiency asymptotically. In other words, the estimate

$$R_\delta \simeq {}^\sharp\mathcal{E}\frac{\bar{\xi}_{QCB}}{\ln 1/\delta}, \tag{13}$$

is exact asymptotically [37]. The close connection between δ and P_e, together with P_e's exponential decay, is responsible for the information deficit appearing only weakly in the redundancy as a logarithm [38]. Equations (11) and (13) are the main results of our Letter. They demonstrate that pure decoherence always gives rise to redundant information, except for cases of measure zero (e.g., when $\rho_k(0) \propto I$ for all k), and give a practical estimate of the redundancy.

Our work connects the physical processes that amplify information with the quantum Chernoff bound. The ratio of the number of copies, R_δ, to the number of subsystems, $^\sharp\mathcal{E}$, of the environment is the efficiency of the copying process: An environment subsystem is imprinted with $\xi_{QCB}/\ln(1/\delta)$ "bits" of information about $\mathcal{S}$. In this sense, ξ_{QCB} is a measure of the efficiency of the amplification: When Nature consumes $^\sharp\mathcal{E}$ environment subsystems – the "raw material" – then $R_\delta \propto \xi_{QCB}{}^\sharp\mathcal{E}$ copies of the system – the final product – are proliferated into the world. Quantum Darwinism, our discussion suggests, can be regarded as a new kind of communication channel – an amplification channel: The same information gets transmitted over and over again, leading to perception of objective reality.

Now let's consider an example: a photon environment decohering a small object initially in a spatial superposition $|\psi_{\mathcal{S}}^0\rangle \propto |\vec{x}_1\rangle + |\vec{x}_2\rangle$ through elastic scattering [3, 17, 25]. We assume the object is heavy enough that its recoil is negligible, and that the wavelength of the light is much longer than the object's extent. This means that the unitary governing the joint evolution of the system and environment is $|\vec{x}_1\rangle\langle\vec{x}_1|\otimes S_{\vec{x}_1} + |\vec{x}_2\rangle\langle\vec{x}_2|\otimes S_{\vec{x}_2}$when restricted to the relevant two-dimensional subspace of $|\psi_{\mathcal{S}}\rangle$, so the Hamiltonian is indeed of the form in Eq. (4). Here, $S_{\vec{x}}$ are the scattering matrices for a single photon scattering off the object at position $\vec{x}$. For thermally distributed radiation which originates from a blackbody covering an arbitrary subset $\mathbb{B}$ of the unit sphere – the "sky" – $\mathbb{S}$ (as viewed from the object), the redundancy of information, deposited in the environment, about the position of the object is calculated in Refs. [13, 26, 27]. The quantum Chernoff information yields that result (up to a factor $1+\ln(2\ln 2)/\ln(\delta)$, which approaches unity as $\delta\to 0$) much more compactly and sheds light on the significance of the different factors that appear within the redundancy.

The photon momentum eigenstates are naturally broken into a tensor product of the magnitude and direction of the momentum. Since the scattering is elastic and recoilless, a photon's interaction with the system can only cause mixing between directional eigenstates inside a subspace of constant energy. This means that the initial thermal mixedness of the photons does not compete with information acquisition. Note, of course, that shorter wavelengths are more efficient at distinguishing between positions of the system, i.e., they have a higher susceptibility. On the other hand we will see that, compared to the case of illumination by a point source [13], the angular spread due to the finite size of $\mathbb{B}$ will make it more difficult to acquire information regarding the position of the object.

If we discretize the photon directional states $|\hat{p}\rangle$ into bits with small solid angle ΔA, the initial state of a blackbody photon k is

$$\rho_k(0) = \int_0^\infty dp\ P(p)|p\rangle\langle p| \otimes \frac{\Delta A}{A_{\mathbb{B}}} \sum_{\hat{p}\in\mathbb{B}} |\hat{p}\rangle\langle\hat{p}| \tag{14}$$

where $A_{\mathbb{B}}$ is the solid angle covered by $\mathbb{B}$. The distribution of energy (momentum magnitude) eigenstates $|p\rangle$ is $P(p) \propto p^2/[\exp(pc/k_BT)-1]$ for some temperature T. Importantly, all the directional eigenstates $|\hat{p}\rangle$ in the support $\mathbb{B}$ are initially equally likely because blackbodies are Lambertian radiators [28, 29]. This means that the initial state in a fixed-p subspace is in the block form

$$\langle p|\rho_k(0)|p\rangle \propto \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} = Q, \tag{15}$$

for projector Q onto the photon directional eigenstates in $\mathbb{B}$. Since the spatial position of the object is only recorded in the direction, not energy, of the outgoing photon, the unitary scattering operator $S_{\vec{x}}$ (conditional on a position $\vec{x}$ of the system) obeys $S_{\vec{x}}(|p\rangle\otimes|\hat{p}\rangle) = |p\rangle\otimes(S^p_{\vec{x}}|\hat{p}\rangle)$, where $S^p_{\vec{x}}$ is the operator restricted to fixed-p subspace. With $Q_{p|\vec{x_i}} = S^p_{\vec{x_i}} Q S^{p\dagger}_{\vec{x_i}}$ for $i = 1, 2$, the trace in Eq. (9) is proportional to

$$\int_0^\infty dp\ P(p)\,\mathrm{Tr}[Q_{p|\vec{x_1}} Q_{p|\vec{x_2}}]. \tag{16}$$

This is independent of c because the $Q_{p|\vec{x_i}}$ are projectors (so $Q^c_{p|\vec{x_i}} = Q^{1-c}_{p|\vec{x_i}} = Q_{p|\vec{x_i}}$ for $c \neq 0, 1$). Thus, this is a case where the optimization over c can be performed.

We consider $^\sharp\mathcal{E}$ photons in a box of volume V, and then take $^\sharp\mathcal{E}, V \to \infty$ while holding the number density $^\sharp\mathcal{E}/V$ fixed to obtain the correct radiation flux. In the position basis, the off-diagonal elements of the density matrix of the object are suppressed by the decoherence factor $\Gamma = \exp(-t/\tau_D)$. The decoherence time τ_D is set by [3, 13, 17, 25]

$$\frac{t}{2\tau_D} = \lim_{^\sharp\mathcal{E}\to\infty} {}^\sharp\mathcal{E}\left(1 - \frac{\Delta A}{A_{\mathbb{B}}} \mathrm{Re} \int_0^\infty dp\ P(p) \sum_{\hat{n}\in\mathbb{B}} \langle\hat{n}|S^{p\dagger}_{\vec{x_1}} S^p_{\vec{x_2}}|\hat{n}\rangle\right). \tag{17}$$

Individual photon momentum eigenstates are diffuse in the $V \to \infty$ limit, so $S^p_{\vec{x}}$ approaches the identity operator. Ignoring higher order terms which disappear in this limit, we find for all c that

$$\begin{aligned}
\ln\left[\mathrm{Tr}(\rho^c_{k|\vec{x_1}} \rho^{1-c}_{k|\vec{x_2}})\right] &\approx \mathrm{Tr}(\rho^c_{k|\vec{x_1}} \rho^{1-c}_{k|\vec{x_2}}) - 1 \\
&= \frac{\Delta A}{A_{\mathbb{B}}} \int_0^\infty dp\ P(p) \sum_{\hat{n}\in\mathbb{B}} \sum_{\hat{m}\in\mathbb{B}} |\langle\hat{n}|S^{p\dagger}_{\vec{x_1}} S^p_{\vec{x_2}}|\hat{m}\rangle|^2 - 1 \\
&= \frac{\Delta A}{A_{\mathbb{B}}} \int_0^\infty dp\ P(p) \sum_{\hat{n}\in\mathbb{B}} \sum_{\hat{m}\in\mathbb{B}} |\langle\hat{n}|(S^{p\dagger}_{\vec{x_1}} S^p_{\vec{x_2}} - I)|\hat{m}\rangle|^2 \\
&\quad - 2\left(1 - \frac{\Delta A}{A_{\mathbb{B}}} \mathrm{Re} \int_0^\infty dp\ P(p) \sum_{\hat{n}\in\mathbb{B}} \langle\hat{n}|S^{p\dagger}_{\vec{x_1}} S^p_{\vec{x_2}}|\hat{n}\rangle\right) \\
&\to -\frac{\alpha}{^\sharp\mathcal{E}} \frac{t}{\tau_D},
\end{aligned} \tag{18}$$

where

$$\alpha = \frac{\int_0^\infty dp\ P(p) \int_{\mathbb{B}} d\hat{n} \int_{\mathbb{S}\backslash\mathbb{B}} d\hat{m} |\langle\hat{n}|(S^{p\dagger}_{\vec{x_1}} S^p_{\vec{x_2}} - I)|\hat{m}\rangle|^2}{\int_0^\infty dp\ P(p) \int_{\mathbb{B}} d\hat{n} \int_{\mathbb{S}} d\hat{m} |\langle\hat{n}|(S^{p\dagger}_{\vec{x_1}} S^p_{\vec{x_2}} - I)|\hat{m}\rangle|^2}, \tag{19}$$

is the so-called *receptivity* of the environment to making records about the system [26]. Its form guarantees that $0 \leq \alpha \leq 1$. We have made use of the definitions of τ_D and $\rho_{k|\vec{x}_i} = S_{\vec{x_i}} \rho_k(0) S^\dagger_{\vec{x_i}}$, the completeness relations $\mathrm{I} = \sum_{\hat{m}\in\mathbb{S}} |\hat{m}\rangle\langle\hat{m}|$, and that $\sum_{\hat{n}\in\mathbb{B}} = \sum_{\hat{n}\in\mathbb{S}} - \sum_{\hat{n}\in\mathbb{S}\backslash\mathbb{B}}$. ($\mathbb{S}\backslash\mathbb{B}$ is the complement set of $\mathbb{B}$ inside $\mathbb{S}$.)

Plugging Eq. (18) into Eq. (13), we obtain

$$R_\delta \simeq \frac{\alpha t/\tau_D}{\ln 1/\delta}, \tag{20}$$

which, as $\delta \to 0$, is Eqs. (24) and (25) from Ref. [26]. Redundant information is thus generated at a rate of $\alpha/\left(\tau_D \ln(1/\delta)\right)$. The factors involved signify three essential ingredients of information: $\ln(1/\delta)$ reflects the accuracy of the information desired by an observer; τ_D represents that the environment and system have interacted which simultaneously decoheres the system and transfers information; α is how receptive the environment is to acquiring information. The *redundancy rate* thus has a remarkably simple and transparent form when evaluated using the quantum Chernoff information.

Conclusions. – We demonstrated how processes that are ubiquitous in the natural world, such as photon illumination, amplify selected information about quantum systems. Photon and photonlike environments give rise to the redundant proliferation of information regarding pointer states – they are the mechanism by which one original becomes many. Information can then be accessed simultaneously and independently by many observers. Objective, classical reality appears as a consequence. The "typical" quantum Chernoff information, $\bar{\xi}_{\mathrm{QCB}}$, quantifies the efficiency of the amplification, which is strictly positive except for measure zero scenarios. The resultant amplification is huge, as it is linear in the environment size, $^\sharp\mathcal{E}\bar{\xi}_{\mathrm{QCB}}/\ln(1/\delta)$. The information disseminated through the environment resides in the states of its individual subsystems. They allow one to acquire the information about the pointer states the "systems of interest" indirectly, via the fragments of $\mathcal{E}$. This amplification and proliferation of selected information results in the emergence of (our perception of) the classical world. The interplay between information available locally from the environment and its complement (quantified by quantum discord) explains the origins of objective reality in the quantum Universe [5, 30, 31] and helps delineate the quantum-classical border.

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[32] In order to have "complete" decoherence, $\hat{\Pi}_{\mathcal{S}}$ and Υ_k should not have degenerate eigenvalues (uniformly for all k).

[33] This inequality is equivalent to $0 \leq P_e^\star + (1-P_e^\star)\ln(1-P_e^\star)$, which can be proven by expanding the logarithm and rearranging the sum to explicitly show that every term is positive.

[34] Implicit in this limit is that the coupling to each environment subsystem does not decay too rapidly as the environment size grows. In the case of the photon environment, this condition is satisfied, as there is a continuous flux of photons that scatter off the system.

[35] Notice that the typical Chernoff information is the quantity of relevance for amplification, as we will show. In the case of hypothesis testing by a single observer, however, one would consider $-\langle \ln \mathrm{tr}\left[\rho_{k|1}^{c}\,\rho_{k|2}^{1-c}\right]\rangle_{k\in\mathcal{E}}$, as we will discuss in a paper in preparation.

[36] Here we use that for i.i.d. states the QCB yields the exact exponent in the asymptotic regime.

[37] When $\bar{\xi}_{\mathrm{QCB}}$ is large, an observer gains a substantial amount of information by intercepting a very tiny fragment of the environment, potentially gaining nearly complete information when acquiring a single environment subsystem. When this occurs, the information deficit, δ, has to be smaller than some maximum, ensuring that $R_\delta/{}^\sharp\mathcal{E}$ is always less than one.

[38] We note that three inequalities were used, which can give a poor bound in some cases of interest. However, due to the connection between δ and P_e, (pre)factors will drop out after taking the logarithm and the asymptotic limit. Thus, the final result can be tight.